Math 253 Calculus III (Bueler) 16 April 2018

Worksheet: Triple integrals in cartesian and spherical coordinates

Recall $dV = dx\,dy\,dz$ in cartesian and $dV = \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$ in spherical coordinates.

A. Suppose E is enclosed by the surfaces $z = x^2 - 1$, $z = 1 - x^2$, $y = 0$, and $y = 2$. Completely set up, but do not evaluate, the triple integral:

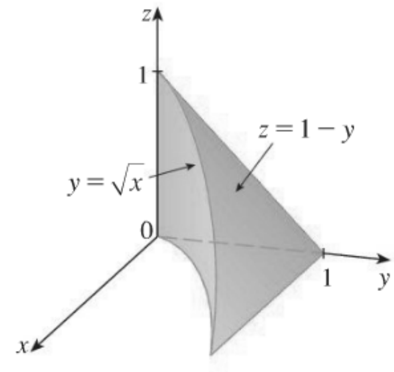
$$\iiint_E (x - y)\, dV$$

B. Suppose E is the region between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$. Choose cartesian or spherical coordinates and then evaluate the integral:

$$\iiint_E (x^2 + y^2)\, dV$$

C. Suppose E is the region shown at right. Completely set up, but do not evaluate, the triple integral

$$\iiint_E f(x, y, z)\, dV$$



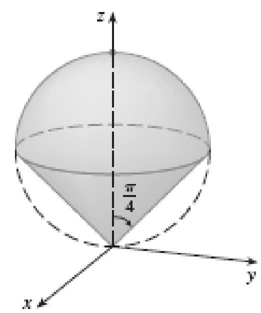
D. The centroid $(\bar{x}, \bar{y}, \bar{z})$ of a three-dimensional object E is computed by integrals for the mass and the three moments assuming constant density K:

$$m = \iiint_E K\, dV,$$

$$M_{yz} = \iiint_E x\, K\, dV, \quad M_{xz} = \iiint_E y\, K\, dV, \quad M_{xy} = \iiint_E z\, K\, dV$$

The centroid is given by these ratios:

$$\bar{x} = \frac{M_{yz}}{m}, \qquad \bar{y} = \frac{M_{xz}}{m}, \qquad \bar{z} = \frac{M_{xy}}{m}.$$



Use spherical coordinates to completely set up, but do not evaluate, the integrals needed to compute the centroid of the region E which is shown. The sphere shown is $x^2 + y^2 + z^2 = z$, which is $\rho = \cos\phi$ in spherical coordinates, and the cone is $z = \sqrt{x^2 + y^2}$.