SOLUTIONS

Math 252 Calculus II (Bueler) 16 April 2018

Worksheet: Series again

For each of the following 10 infinite series, state whether it *converges absolutely*, *converges conditionally*, or *diverges*. If a parameter appears (e.g. "x" or "r") then give the answer for all cases of that parameter. Justify your statement using the following tests and categories:

- test for divergence
- geometric series
- integral test
- p-series
- comparison test
- limit comparison test
- alternating series test
- ratio test
- root test

Show appropriate work when applying a test. Multiple tests may apply; focus on successfully applying the easiest test that does the job.

A. $$\sum_{n=0}^{\infty} \frac{1}{n!}$$

converges absolutely

ratio test: $L<1$

$$L = \lim_{n\to\infty} \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} = \lim_{n\to\infty} \frac{n!}{(n+1)!} = \lim_{n\to\infty} \frac{1}{n+1} = 0$$

B. $$\sum_{n=1}^{\infty} x^n$$

converges absolutely for $|x|<1$

diverges for all other x

geometric series with $a=x$, $r=x$

C. $$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

converges absolutely

ratio test: $L<1$

$$L = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{(n+1)2^n}{n\,2^{n+1}} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2}$$

D. $$\sum_{n=1}^{\infty} \left(\frac{n}{-3}\right)^n$$

diverges

root test: $L>1$

$$L = \lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{n}{-3}\right)^n\right|} = \lim_{n\to\infty} \frac{n}{3} = +\infty$$

E.

$$\sum_{n=7}^{\infty} \frac{(-1)^n \ln n}{n}$$

converges conditionally

alt. series test: $b_n = \frac{\ln n}{n} \to 0$, decreases

$\sum_{n=7}^{\infty} \frac{\ln n}{n}$ diverges (comparison or integral test)

F.

$$\sum_{n=1}^{\infty} \frac{n}{3^n + 5}$$

converges absolutely:

comparison test: $a_n = \frac{n}{3^n+5} \le \frac{n}{3^n} = b_n$

and ratio test: $L = \lim_{n\to\infty} \frac{b_{n+1}}{b_n} = \lim_{n\to\infty} \frac{(n+1)3^n}{n\,3^{n+1}} = \lim_{n\to\infty} \frac{n+1}{3n} = 0$

$L < 1$

G.

$$\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2}$$

converges absolutely:

comparison test: $|a_n| = \frac{|\sin n|}{n^2} \le \frac{1}{n^2} = b_n$

and $\sum b_n$ is $p=2$ series

H.

$$\sum_{k=1}^{\infty} \frac{r^k}{k!}$$

converges absolutely for all r

ratio test: $L = \lim_{k\to\infty} \frac{\frac{|r|^{k+1}}{(k+1)!}}{\frac{|r|^k}{k!}} = \lim_{k\to\infty} \frac{|r|^{k+1} k!}{|r|^k (k+1)!} = \lim_{k\to\infty} \frac{|r|}{k+1}$

$L < 1$

$= 0$

I.

$$\sum_{n=1}^{\infty} \frac{2n}{n^2 - 3}$$

diverges:

limit comparison test: $a_n = \frac{2n}{n^2-3}$, $b_n = \frac{1}{n}$, $\sum b_n$ $p=1$ series

$c = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2n(n)}{n^2-3} = 2$, $0 < c < \infty$

J.

$$\sum_{n=0}^{\infty} \frac{(-3)^n}{(2n+1)!}$$

converges absolutely:

ratio test:

$L = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{3^{n+1}}{(2n+3)!} \frac{(2n+1)!}{3^n} = \lim_{n\to\infty} \frac{3}{(2n+3)(2n+2)}$

$= 0$

$L < 1$