Worksheet: Power series

1. Find the radius and interval of convergence:

$$\sum_{n=0}^{\infty} \frac{(3x+2)^n}{n!}$$

2. Find the radius and interval of convergence:

$$\sum_{n=1}^{\infty} n(x-7)^n$$

3. Find the radius and interval of convergence:

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{3^n}$$

- 4. The goal here is to accurately do an integral, by using power series, that we could not do before.
- (a) Compute the sum of the series assuming |x| < 1:

$$\sum_{n=0}^{\infty} x^n =$$

(b) Substitute $-x^4$ for x to get a power series for this function:

$$\frac{1}{1+x^4} =$$

(c) Integrate term-by-term to get a power series:

$$\int \frac{1}{1+x^4} \, dx =$$

- (d) What is the radius and interval of convergence of the above series?
- **(e)** Evaluate to get a series (*note x is gone so it is no longer a power series*!):

$$\int_0^{0.2} \frac{1}{1+x^4} \, dx =$$

- (f) How many terms are needed to get the integral in (e) to within 10^{-6} ? Why?
- (g) Approximate to with 10^{-6} . Only this part might need a calculator:

$$\int_{0}^{0.2} \frac{1}{1+x^4} dx \approx$$