18 Integrals: Solutions

Now that you have done your share of the work, here are all answers together in one place. Problems labeled as HARD or IMPOSSIBLE are types that will not be on quizzes and exams. Of course, HARD problems will be on Written Homework and WebAssign.

A. Compare **Q**. *integration-by-parts*

$$\int xe^{3x}\,dx = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

B. Use $x = 2\tan\theta$ and then use result from **H** below. *trigonometric substitution*, HARD

$$\int_2^{2\sqrt{3}} \sqrt{x^2+4}\,dx = 2\left(2\sqrt{3} - \sqrt{2} + \ln(2+\sqrt{3}) - \ln(1+\sqrt{2})\right)$$

C. Use $u = \cos 5\theta$. *u-substitution*

$$\int_0^{\pi} \sin 5\theta \cos^3 5\theta\,d\theta = 0$$

D. Use $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ etc. *trigonometric integral*

$$\int \sin^2 x \cos^2 x\,dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$$

E. Use $u = 5x^2$. Compare **L**. *u-substitution*

$$\int x\cos(5x^2)\,dx = \frac{1}{10}\sin(5x^2) + C$$

F. Observe cancellation. *trigonometric integral*

$$\int \sin t \cot t\,dt = \sin t + C$$

G. Use $u = \tan\theta$. *u-substitution*

$$\int_0^{\pi/4} \tan^2\theta \sec^2\theta\,d\theta = \frac{1}{3}$$

H. Factor $\sec^2 x$, do integration-by-parts, and get equation "$I = [\text{known}] - I$".
trigonometric integral, *integration-by-parts*, HARD

$$\int \sec^3\theta\,d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

I. Use $2\sin\theta\cos\theta = \sin 2\theta$ and $u = \cos x$. *trigonometric integral*, *u-substitution*

$$\int \cos(x)\sin(2x)\,dx = -\frac{2}{3}\cos^3 x + C$$

J. Use $u = x^2$. *u-substitution*

$$\int \frac{x\,dx}{1+x^4} = \frac{1}{2}\arctan(x^2) + C$$

K. Integration-by-parts then $u = 1 - 4\theta^2$. *integration-by-parts, u-substitution*

$$\int \arccos(2\theta)\,d\theta = \theta\arccos(2\theta) - \frac{1}{2}\sqrt{1-4\theta^2} + C$$

L. Compare **E**. IMPOSSIBLE (using familiar functions)

$$\int x^2\cos(5x^2)\,dx = ??$$

M. Integration-by-parts twice to get equation "$I = [\text{known}] - 4I$". *integration-by-parts*

$$\int e^{2x}\sin x\,dx = \frac{1}{5}e^{2x}\left(2\sin x - \cos x\right) + C$$

N. Complete square then substitution $x - 1 = 3\sin\theta$. *trigonometric integral*

$$\int \frac{dx}{\sqrt{8+2x-x^2}} = \arcsin\left(\frac{x-1}{3}\right) + C$$

O. Factor $\sec\theta\tan\theta$ for du and use $u = \sec\theta$. *trigonometric integral, u-substitution*

$$\int \tan\theta\sec^7\theta\,d\theta = \frac{1}{7}\sec^7\theta + C$$

P. Integration-by-parts. *integration-by-parts*

$$\int x^4\ln x\,dx = \frac{1}{25}x^5\left(5\ln x - 1\right) + C$$

Q. Use $u = 3x^2$. Compare **A**. *u-substitution*

$$\int xe^{3x^2}\,dx = \frac{1}{6}e^{3x^2} + C$$

R. Use $x = \sec\theta$ and triangle to convert limits. *trigonometric integral*

$$\int_1^7 \frac{\sqrt{x^2-1}}{x}\,dx = \int_0^{\arccos(1/7)} \tan^2\theta\,d\theta = \tan(\arccos(1/7)) - \arccos(1/7)$$
