

Written Homework #8**Due at start of class Monday, 19 March.**

This Written Homework has problems from sections 9.1 and 10.1. It is also a work sheet to do during the recitation section. Please work on it with other students! The submitted version must be written by you. You must show your work for full credit.

1.
 - (a) For what values of k does $y = \cos kt$ satisfy the differential equation $4y'' = -25y$?

 - (b) For those values of k , verify that every member of the family of functions $y = A \sin kt + B \cos kt$ is also a solution.

2. An equation in biology states that the rate of growth of the length L of an individual fish is proportional to the difference between L and the asymptotic length L_∞ .
 - (a) Write a differential equation which expresses this idea.

 - (b) At right, make a sketch, with t and L axes, of the graph of three or four typical solutions of this differential equation, corresponding to different (reasonable) initial values $L(0)$, but with the same L_∞ and constant of proportionality.

3. Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

$$x = e^{-t} + t, \quad y = e^t - t, \quad -2 \leq t \leq 2$$

4. (a) Eliminate the parameter to find a Cartesian equation of the curve.

$$x = \frac{1}{2} \cos \theta, \quad y = 2 \sin \theta, \quad 0 \leq \theta \leq \pi$$

(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

5. Consider the formulas

$$x(t) = \sin(nt), \quad y(t) = \cos(t)$$

where n is a positive integer.

(a) Explain why such a parametric curve is a closed curve. (*Hint.* Are the functions periodic?)

(b) Use a computer to help sketch the curves when $n = 1, 2, 3, 6$. (*Thus show four curves on separate axes below.*) What will the curve be like as $n \rightarrow \infty$?