Math 252 Calculus II (Bueler) Name: ______________________

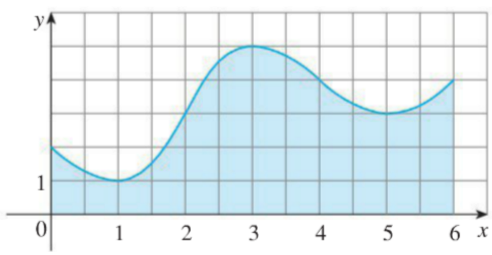
Written Homework #5

Due at start of class Monday, 19 February.

This Written Homework has problems from sections 7.5, 7.7, and 7.8. And it is pretty brief. It is also a "work sheet" to do during the recitation section. Please work on it with other students! The submitted version must be written by you. You must show your work for full credit.

Problems 2 and 3 are reasonably-hard definite integrals from section 7.5. They *can* be done by hand. However, they are good examples of integrals for which one wants a "second opinion" from an independent source. So we do the same integral numerically and check that the results agree to a reasonable degree of precision. The ideas in section 7.7 of the textbook should help mold your expectations of how much agreement to expect. *Read* sections 7.5 and 7.7.

1. Estimate the area under the graph in the figure by using (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson's rule, each with $n = 6$ subintervals.



2. (a) Evaluate the integral by hand:

$$\int_0^1 x\sqrt{2-\sqrt{1-x^2}}\,dx$$

(*Hint: Start doing u-substitutions.*)

(b) Sketch the graph of $f(x) = x\sqrt{2-\sqrt{1-x^2}}$ on $[0,1]$. Use a computer to help you.

(c) Approximate the integral by using the trapezoid rule with $n = 5$ subintervals. What is the error E_T? Will the trapezoid rule give a value that is too large or too small?

3. (a) Evaluate the integral by hand:

$$\int_1^5 (x + \sin x)^2 \, dx$$

(b) Sketch the graph of $g(x) = (x + \sin x)^2$ on $[1, 5]$. Use a computer to help you.

(c) Approximate the integral by using Simpson's rule with $n = 4$ and $n = 8$ subintervals. What is the error E_S in each case?

4. Find all the real values of p for which the integral converges, and evaluate it in those cases:
$\int_0^1 \frac{1}{x^p}\,dx$

5. A radioactive substance decays exponentially so its mass at time t is $m(t) = m(0)e^{-kt}$ where $m(0)$ is the initial mass and k is a positive constant. The *mean life* M of an atom of the substance is

$$M = k \int_0^{\infty} te^{-kt}\,dt.$$

(*This is an* expected value *calculation. The exponential actually represents a probability.*) For the radioactive isotope of carbon ^{14}C, used in "radiocarbon dating," the value of k is 0.000121 and we measure t in years. Find the mean life of a ^{14}C atom.