Math 252 Calculus II (Bueler) Name: ______________________________

Written Homework #14

Due, at latest, in my Chapman 101 office box at 12 noon on Tuesday 1 May 2018.

This Written Homework has problems from sections 11.10 and 11.11. Please work on it with other students! The submitted version must be written by you. You must show your work for full credit.

1. Find the Maclaurin series for $f(x)$ using the definition. (*You may assume f has a power series expansion. You do not need to show that $R_n \to 0$.*) Also find the radius of convergence.

(a)

$$f(x) = 2^x$$

(b)

$$f(x) = x \sin x$$

2. Find the Taylor series for $f(x)$ centered at the given value of a, by using the definition. (*You may assume f has a power series expansion. You do not need to show that $R_n \to 0$.*) Also find the radius of convergence.

(a)

$$f(x) = \ln x, \quad a = 2$$

(b)

$$f(x) = e^{2x}, \quad a = 3$$

(c)

$$f(x) = x^4 - x^3 + 2, \quad a = -2$$

3. (a) Find the Maclaurin series of f (by any method) and its radius of convergence.

$f(x) = \ln(1 + x^2)$

(b) Graph $f(x)$ and its first two distinct Taylor polynomials, namely $T_2(x)$ and $T_4(x)$, on the same axes. Label appropriately. (*Remember that "$T_n(x)$" is a polynomial of degree at most n. Use a computer as needed.*)

4. (a) Approximate f by a Taylor polynomial $T_n(x)$ with degree n at the number a:

$$f(x) = \frac{1}{x}, \quad a = 1, \quad n = 2$$

(b) Now use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x is in the interval $0.7 \leq x \leq 1.3$.

5. (a) Approximate f by a Taylor polynomial $T_n(x)$ with degree n at the number a:

$$f(x) = e^{-x^2}, \quad a = 0, \quad n = 2$$

(b) Now use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x is in the interval $0 \leq x \leq 0.2$.

6. Consider the approximation

$$\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}.$$

(a) Use the Alternating Series Estimation Theorem to estimate the range of values of x for which the given approximation is accurate to within 0.005.

(b) Use Taylor's Inequality to do the same job as in part **(a)**.