Name:

## Written Homework #11

## Due at start of class Monday, 9 April 2018.

This Written Homework has problems from sections 11.3 and 11.4. It is also a work sheet to do during the recitation section. Please work on it with other students! The submitted version must be written by you. You must show your work for full credit.

1. Determine whether the series is convergent or divergent.

$$\frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \dots$$

2. Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$$

**3.** Find the values of *p* for which the series is convergent.

$$\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^p}$$

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- **4.** Leonard Euler was able to calculate the exact sum of the *p*-series with p = 2:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

Use this fact to find the sum of these series:

(a)

$$\sum_{n=2}^{\infty} \frac{1}{n^2} =$$

(b)

$$\sum_{n=3}^\infty \frac{1}{(n+1)^2} =$$

(c)

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} =$$

5. Draw a carefully-labeled picture to show that

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} > \int_{2}^{\infty} \frac{dx}{\sqrt{x}}$$

By computing the improper integral, conclude that the series diverges. (*Hint*. This problem merely asks you to illustrate a case of the Integral Test.)

**6.** Use the Remainder Estimate for the Integral Test to find the sum of the series correct to four decimal places:

$$\sum_{n=1}^{\infty} n e^{-2n}$$

(*Hint*. Draw a picture of the series, and the remainder estimate, to help guide your work.)

7. Show that if  $a_n > 0$  and  $\sum a_n$  is convergent then  $\sum \ln(1 + a_n)$  is convergent.

**8.** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$$

**9.** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^n}$$

**10.** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$$