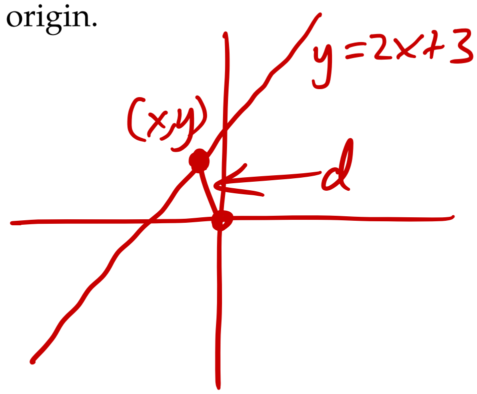
1. Find the point on the line $y = 2x + 3$ which is closest to the origin.



$$d^2 = x^2 + y^2 = x^2 + (2x+3)^2$$

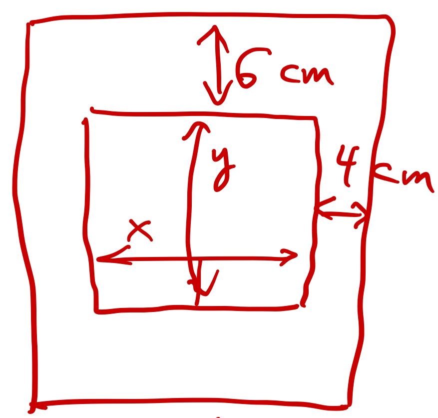
$$f(x) = x^2 + (2x+3)^2$$

$$f'(x) = 2x + 2(2x+3)\cdot 2$$

$$= 10x + 12 = 0$$

$$x = \frac{-6}{5}, \quad y = 2x+3 = \frac{3}{5}$$

2. The top and bottom margins of a poster are each 6 cm and the side margins are 4 cm. If the area of the printed material on the poster is fixed at 384 cm^2, find the dimensions of the poster with the smallest total area.



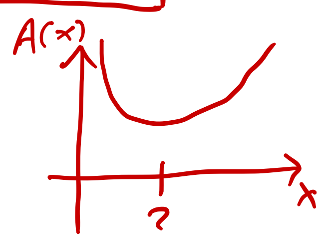
$$xy = 384$$

$$A = (\text{total poster area})$$

$$= (x+8)(y+12)$$

$$= \overbrace{xy}^{=384} + 8y + 12x + 96$$

$$= 12x + 8y + 480 \quad \left(y = \frac{384}{x}\right)$$



$$A(x) = 12x + \frac{3072}{x} + 480$$

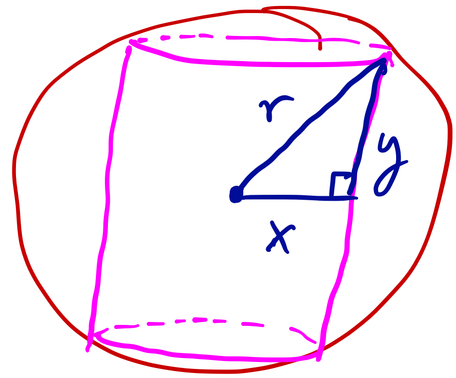
$$A'(x) = 12 - \frac{3072}{x^2} = 0$$

$$x^2 = \frac{3072}{12} = 256$$

$$x = 16 \quad \therefore y = \frac{384}{16} = 24 \quad \therefore$$

dimensions:
width = 24 cm
height = 36 cm

3. A right circular cylinder is inscribed in a sphere of radius r. Find the largest possible volume of such a cylinder.



$$r^2 = x^2 + y^2$$

$$V_{cyl} = \pi x^2 \cdot 2y$$

$$x^2 = r^2 - y^2$$

x = radius of cylinder

$2y$ = height of cylinder

$\therefore$

$$V(y) = \pi (r^2 - y^2) \cdot 2y$$

$$= 2\pi r^2 y - 2\pi y^3$$

$$V'(y) = 2\pi r^2 - 6\pi y^2 = 0$$

$$y^2 = \frac{2\pi r^2}{6\pi} = \frac{1}{3} r^2$$

$$y = +\sqrt{\frac{1}{3}}\, r$$

y	$V(y)$
0	0
$\frac{1}{\sqrt{3}} r$	
r	0

$$V\left(\frac{1}{\sqrt{3}} r\right) = 2\pi r^2 \cdot \frac{1}{\sqrt{3}} r - 2\pi \left(\frac{1}{\sqrt{3}} r\right)^3$$

$$= 2\pi r^3 \left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}\right) = \frac{4\pi}{3\sqrt{3}} r^3$$