SOLUTIONS

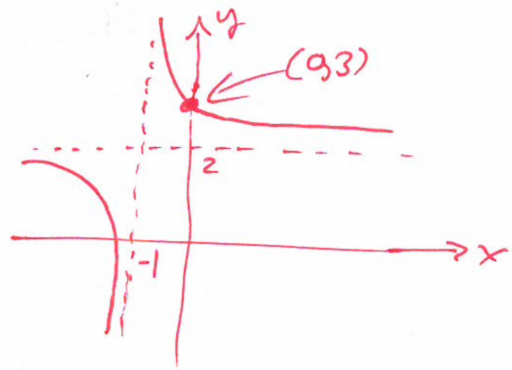
1. (*See examples and exercises in* §2.2 and §2.6.) Give an example of a graph $y = f(x)$ with a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = 2$.

$$f(x) = \frac{1}{x+1} + 2 = \frac{1+2(x+1)}{x+1}$$

raise to get horizontal

(has $x=-1$ vertical but $y=0$ horizontal)

$$y = f(x) = \frac{2x+3}{x+1}$$



2. (*See* §3.4.) Build an example of a complicated chain rule derivative question. That is, write down $f(x)$ and compute the derivative $f'(x)$.

$$f(x) = \ln(x + \arctan x)$$

$$f'(x) = \frac{1}{x + \arctan x}\left(1 + \frac{1}{1+x^2}\right)$$

3. (*See* §5.5.) Write the previous example as an indefinite integration question. Give a substitution which will solve it. Complete the integration.

$$\int \frac{1 + (x^2+1)^{-1}}{x + \arctan x}\,dx = \int \frac{du}{u}$$

$$\left[\begin{array}{l} u = x + \arctan x \\ du = \left(1 + \frac{1}{1+x^2}\right)dx \end{array}\right]$$

$$= \ln|x + \arctan x| + C$$

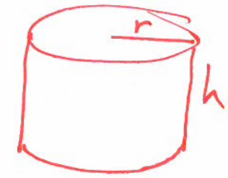
(abs. values are appropriate here)

Some advice for the actual Final Exam:

Read the question. Don't just guess it is of a certain type.

4. (*See* §4.7.) A steel cylindrical can is to hold 1 L of oil. Find the dimensions of the can that will minimize the amount of steel.

measure distances in cm

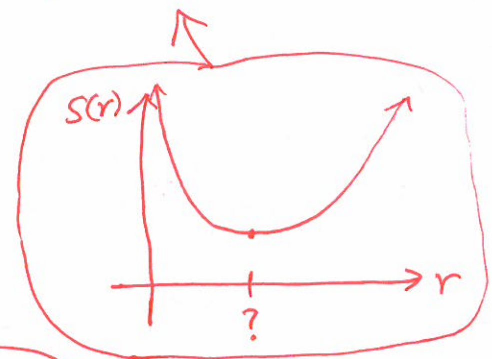


$$V = \pi r^2 h \Rightarrow \underbrace{1000}_{[\mathrm{cm}^3]} = \pi r^2 h \qquad h = \frac{1000}{\pi r^2}$$

$$S = \underbrace{2\pi r h}_{\text{sides}} + \underbrace{2\pi r^2}_{\text{top \& bottom}}$$

$$S(r) = 2\pi r \cdot \frac{1000}{\pi r^2} + 2\pi r^2 = \frac{2000}{r} + 2\pi r^2$$

$$S'(r) = -2000\, r^{-2} + 4\pi r = 0$$



$$\frac{2000}{r^2} = 4\pi r$$

$$\frac{500}{\pi} = \frac{1000}{2\pi} = r^3$$

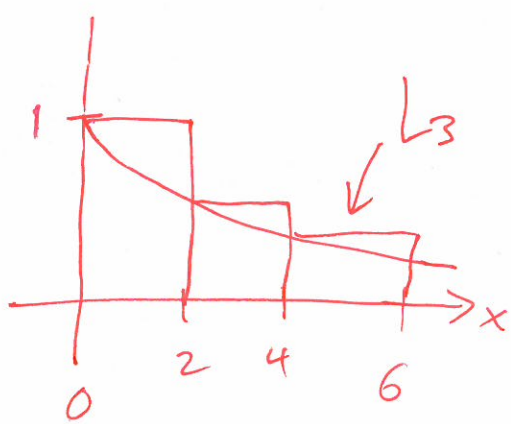
$$\boxed{r = \sqrt[3]{\frac{500}{\pi}}, \quad h = \frac{1000}{\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2} = 2\sqrt[3]{\frac{500}{\pi}}}$$

in cm

5. (*See* §5.1 and §5.2.) For the integral $\int_0^6 \frac{1}{1+x^4}\,dx$, compute the Riemann sums with $n = 3$ rectangles and both left and right endpoints.

$$L_3 = \frac{1}{1+0}\cdot 2 + \frac{1}{1+2^4}\cdot 2 + \frac{1}{1+2^4}\cdot 2$$

$$= 2 + \frac{2}{17} + \frac{2}{33}$$



$$R_3 = \frac{1}{1+2^4}\cdot 2 + \frac{1}{1+4^4}\cdot 2 + \frac{1}{1+6^4}\cdot 2$$

$$= \frac{2}{17} + \frac{2}{33} + \frac{2}{65}$$

6. (*See* §4.3 and §4.5.) Find the critical points, intervals of increase and decrease, and points of inflection of $f(x) = x^3 - 3x - 1$. Then sketch the graph $y = f(x)$.

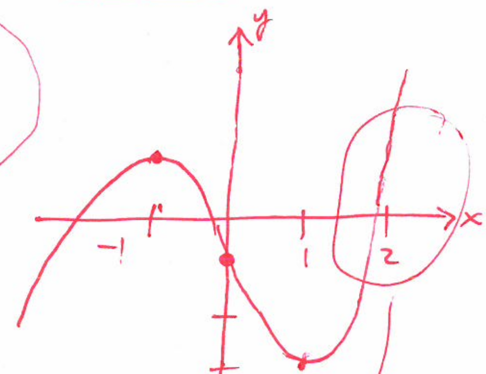
$f'(x) = 3x^2 - 3$ $\qquad f'(x) = 0 \iff x = \pm 1$ critical #s

$f''(x) = 6x$ $\qquad f''(x) = 0 \iff x = 0$ inflection pt

x	f	f'
		$+$
-1	1	0
0	-1	$-$
$+1$	-3	0
		$+$

increasing $(-\infty, -1) \cup (+1, \infty)$

decreasing $(-1, 1)$



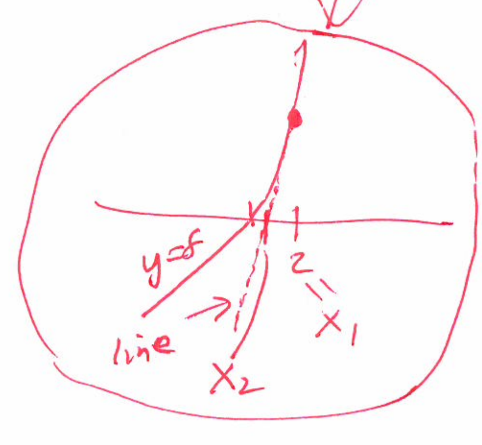
7. (*See* §4.8.) In the graph above there is a solution of $f(x) = 0$ near $x = 2$. Approximate it using one step of Newton's method, and add that to your sketch.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 2$$

$$x_2 = x_1 - \frac{x_1^3 - 3x_1 - 1}{3x_1^2 - 3}$$

$$= 2 - \frac{8 - 6 - 1}{12 - 3} = 2 - \frac{1}{9}$$



8. (*See* §3.5.) Find dy/dx by implicit differentiation: $\qquad y \cos x = x^2 + y^2$

$$\frac{dy}{dx} \cos x + y(-\sin x) = 2x + 2y \frac{dy}{dx}$$

$$\frac{dy}{dx}(\cos x - 2y) = 2x + y \sin x$$

$$\frac{dy}{dx} = \frac{2x + y \sin x}{\cos x - 2y}$$