SOLUTIONS

Math F251: Another Section 5.4 and 5.5 Worksheet

Table of Indefinite Integrals

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\int k\,dx = kx + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ (n \neq -1)$ $\quad$ $\int \frac{1}{x}\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$ $\quad$ $\int b^x\,dx = \frac{b^x}{\ln b} + C$

$\int \sin x\,dx = -\cos x + C$ $\quad$ $\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$ $\quad$ $\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$ $\quad$ $\int \csc x \cot x\,dx = -\csc x + C$

$\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$ $\quad$ $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

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1. For the following integrals, decide if you would use a u-substitution. If so, *just write down the u-substitution*. If not, *evaluate the integral*.

(a) $\int e^{\cos x} \sin x\,dx =$ $\quad$ $u = \cos x$

(b) $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$ $\quad$ (or $u = ax+b$)

(c) $\int_0^2 |2x-1|\,dx = 5/2$ $\quad$ ← see explanation on back

(d) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} =$ $\quad$ $u = \ln x$

(e) $\int (7x - 7^{-x})\,dx = \frac{7}{2}x^2 + 7^{-x} + C$ $\quad$ ← by guess-and-check

(f) $\int_0^1 x(\sqrt[3]{x} + \sqrt[4]{x})\,dx = \int_0^1 x^{4/3} + x^{5/4}\,dx = \left.\frac{3}{7}x^{7/3} + \frac{4}{9}x^{9/4}\right]_0^1 = \frac{3}{7} + \frac{4}{9} = \frac{55}{63}$

(g) $\int \pi\,dt = \pi t + C$

(h) $\int \frac{3\,dr}{\sqrt{1-r^2}} = 3\arcsin(r) + C$

(i) $\int \tan^2\theta \sec^2\theta\,d\theta =$ $\quad$ $u = \tan\theta$

(j) $\int \frac{dx}{(1+x^2)\tan^{-1}(x)} =$ $\quad$ $u = \tan^{-1}(x)$

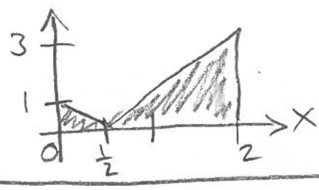
2. Complete the u-substitution, or any other work, for the integrals from problem **1.**

(a) $= \int e^u(-du) = -e^u + c = -e^{\cos x} + c$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

(b) $= \int \frac{du/a}{u} = \frac{1}{a}\int \frac{du}{u} = \frac{1}{a}\ln|u| + c$
$= \frac{1}{a}\ln|ax+b| + c$

$u = ax+b$
$du = a\,dx$
$du/a = dx$

(c) 

$\leftarrow$ total area $= \frac{1}{2}\cdot\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{3}{2}\cdot 3 = \frac{10}{4} = \frac{5}{2}$

(d) $= \int_1^4 \frac{du}{\sqrt{u}} = \int_1^4 u^{-1/2}\,du$

$u = \ln x$
$du = \frac{1}{x}\,dx$

(e) $= 2u^{1/2}\Big]_1^4 = 2(\sqrt{4} - \sqrt{1}) = 6$

$u(e) = \ln e = 1$
$u(e^4) = \ln(e^4) = 4$

(f)

(g)

(h) $= \int u^2\,du = \frac{1}{3}u^3 + C$
$= \frac{1}{3}\tan^3\theta + C$

$u = \tan\theta$
$du = \sec^2\theta\,d\theta$

(i)

(j) $= \int \frac{du}{u} = \ln|u| + c$
$= \ln|\tan^{-1} x| + C$

$u = \tan^{-1} x$
$du = \frac{1}{1+x^2}\,dx$