1. Evaluate the integral by making the given substitution.

(a) $u = \sin\theta$:

$$\int \sin^2\theta\cos\theta\,d\theta = \int u^2\,du = \frac{1}{3}u^3 + C$$

$$\left[\begin{array}{l} u = \sin\theta \\ du = \cos\theta\,d\theta \end{array}\right]$$

$$= \frac{1}{3}\sin^3\theta + C$$

(b) $u = x^4 - 5$:

$$\int \frac{x^3}{x^4-5}\,dx = \int \frac{du/4}{u} = \frac{1}{4}\int \frac{du}{u}$$

$$\left[\begin{array}{l} u = x^4-5 \\ du = 4x^3dx \\ \frac{du}{4} = x^3dx \end{array}\right]$$

$$= \frac{1}{4}\ln|u| + C$$

$$= \frac{1}{4}\ln|x^4-5| + C$$

2. Evaluate the indefinite integral by substitution. What should you choose as u?:

$$\int e^x\sqrt{1+e^x}\,dx = \int \sqrt{u}\,du = \int u^{1/2}du$$

$$\left[\begin{array}{l} u = 1+e^x \\ du = e^x dx \end{array}\right]$$

$$= \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{3}(1+e^x)^{3/2} + C$$

3. Evaluate the indefinite integrals:

(a)

$$\int 5^t \sin(5^t)\,dt =$$

$$\begin{bmatrix} u = 5^t \\ du = 5^t \cdot \ln 5\, dt \\ \frac{du}{\ln 5} = 5^t dt \end{bmatrix}$$

$$\int \sin u \frac{du}{\ln 5} = \frac{1}{\ln 5}\int \sin u\, du$$

$$= \frac{1}{\ln 5}(-\cos u) + C$$

$$= -\frac{1}{\ln 5}\cos(5^t) + C$$

(b)

$$\int \frac{x}{1+x^4}\,dx =$$

$$\begin{bmatrix} u = x^2 \\ du = 2x\,dx \\ \frac{du}{2} = x\,dx \end{bmatrix}$$

$$\int \frac{du/2}{1+u^2} = \frac{1}{2}\int \frac{du}{1+u^2}$$

$$= \frac{1}{2}\arctan(u) + C$$

$$= \frac{1}{2}\arctan(x^2) + C$$

(c)

$$\int (3t-1)^{50}\,dt =$$

$$\begin{bmatrix} u = 3t-1 \\ du = 3\,dt \\ \frac{du}{3} = dt \end{bmatrix}$$

$$\int u^{50}\frac{du}{3} = \frac{1}{3}\frac{u^{51}}{51} + C$$

$$= \frac{1}{153}(3t-1)^{51} + C$$

(d)

$$\int \cos x \,\sin(\sin(x))\,dx =$$

$$\begin{bmatrix} u = \sin x \\ du = \cos x\,dx \end{bmatrix}$$

$$\int \sin(u)\,du$$

$$= -\cos(u) + C$$

$$= -\cos(\sin x) + C$$