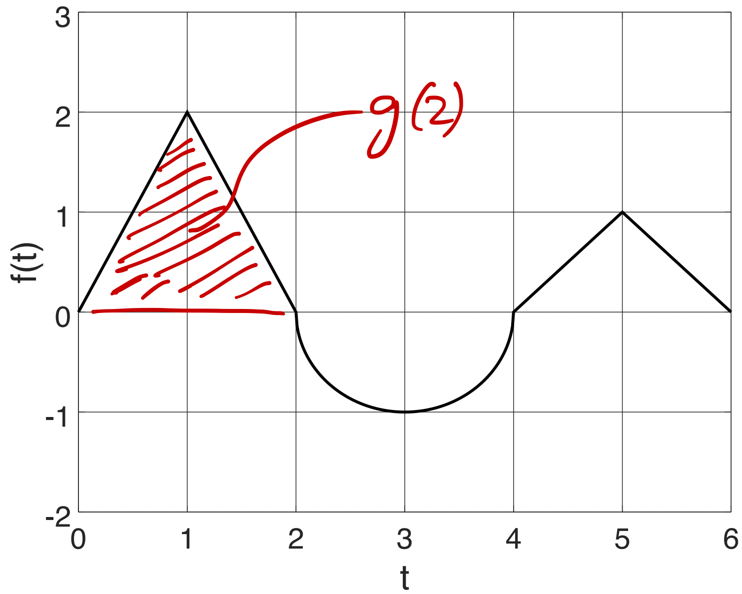
1. (a) The graph of $f(t)$ is at right. Suppose we define the new function

$$g(x) = \int_0^x f(t)\,dt$$

Sketch $g(2)$ as an area. ✓



(b) What are the exact values of $g(0), g(2), g(4), g(6)$? (*Assume the curved part is circular.*)

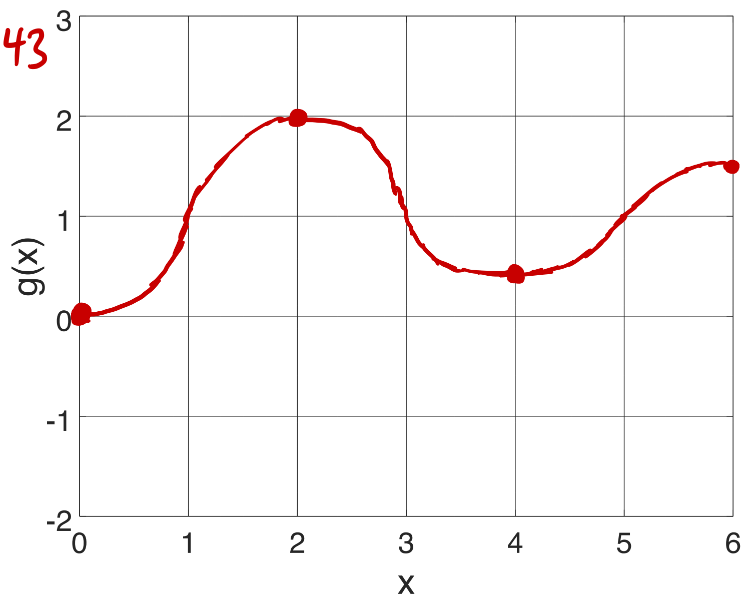
$g(0) = 0$

$g(2) = 2$

$g(4) = 2 - \frac{\pi}{2} \approx 0.43$

$g(6) = 2 - \frac{\pi}{2} + 1 = 3 - \frac{\pi}{2} \approx 1.43$

(c) Sketch the graph of $g(x)$ on the axes. →



(d) What is the graph of $g'(x)$?

$g'(x) = f(x)$

2. Evaluate the integral:

$$\int_0^1 (1+r)^3\,dr = \frac{1}{4}(1+r)^4\Big]_0^1 = \frac{1}{4}\left((1+1)^4 - (1+0)^4\right)$$

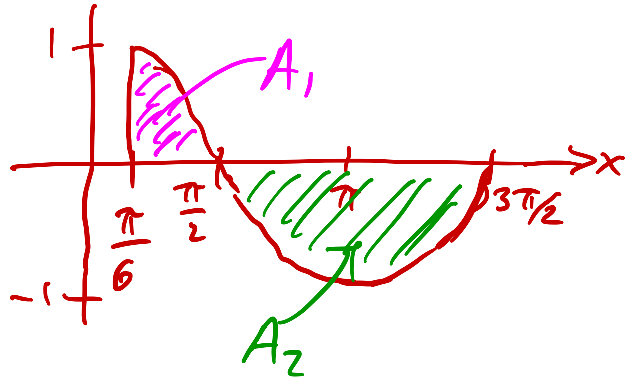
$$= \frac{1}{4}(16-1) = \frac{15}{4}$$

3. Evaluate the integral and interpret as a difference of areas:

$$\int_{\pi/6}^{3\pi/2} \cos x\, dx = \sin x \Big]_{\pi/6}^{3\pi/2} = \sin\left(\frac{3\pi}{2}\right) - \sin\left(\pi/6\right)$$

$$= (-1) - \left(\frac{1}{2}\right) = -\frac{3}{2}$$

$$= A_1 - A_2$$



4. Evaluate the integral:

$$\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}\, dx = 8\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2}\, dx$$

$$= 8 \arctan(x)\Big]_{1/\sqrt{3}}^{\sqrt{3}} = 8\arctan(\sqrt{3}) - 8\arctan(1/\sqrt{3})$$

$$= 8\cdot \pi/3 - 8\cdot \pi/6 = \frac{4\pi}{3}$$

5. (a) Use part II of the Fundamental Theorem of Calculus to compute

$$y(x) = \int_{\cos x}^{\pi} \theta^2\, d\theta$$

Then differentiate to find dy/dx.

$$y(x) = \frac{1}{3}\theta^3\Big]_{\cos x}^{\pi} = \frac{1}{3}\pi^3 - \frac{1}{3}(\cos x)^3$$

$$\therefore \quad \frac{dy}{dx} = 0 - (\cos x)^2 \cdot (-\sin x)$$

$$= +\cos^2 x \sin x$$

(b) Use part I of the Fundamental Theorem of Calculus, and the chain rule, to find dy/dx ... and get the same result as in (a).

$$\frac{dy}{dx} = \frac{d}{dx}\left(\int_{\cos x}^{\pi} \theta^2 d\theta\right) = -\frac{d}{dx}\left(\int_{\pi}^{\cos x} \theta^2 d\theta\right)$$

$$= -\underbrace{(\cos x)^2}_{\text{FTC I}} \cdot \underbrace{(-\sin x)}_{\text{chain rule}} = +\cos^2 x \sin x$$