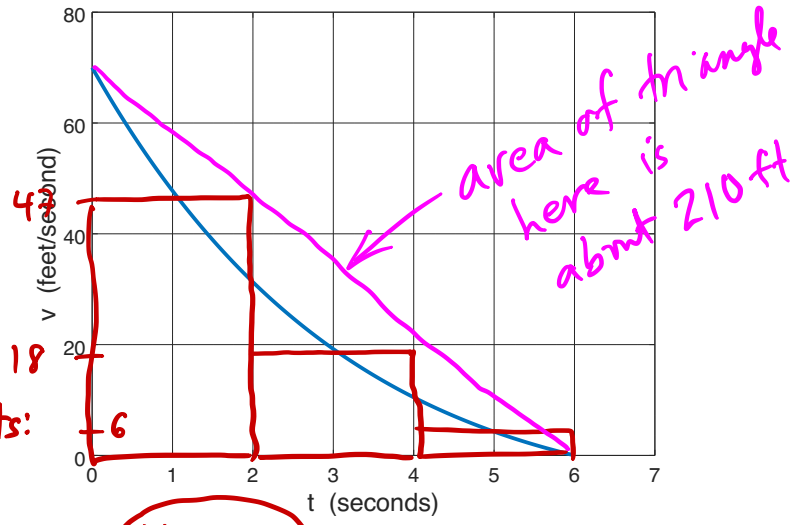
SOLUTIONS

1. The velocity graph $v(t)$ of a braking car is shown.

(a) Use the graph to estimate the distance traveled by the car when the brakes are applied. (*Suggestion: Use 3 or 6 rectangles.*)



[your results will vary ... but your answer should be 100 – 200 ft]

with n=3 rectangles and midpoints:

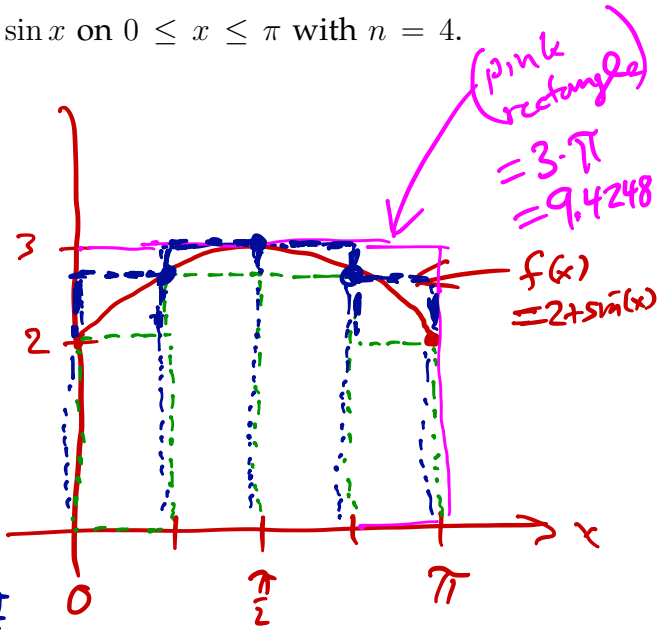
$$d \approx 47 \cdot 2 + 18 \cdot 2 + 6 \cdot 2 = 142 \text{ ft}$$

(b) Write the exact distance as a definite integral.

$$\int_0^6 v(t)\,dt$$

2. Evaluate the upper and lower sums for $f(x) = 2 + \sin x$ on $0 \le x \le \pi$ with $n = 4$. Illustrate with a diagram.

$$\Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$$



(lower sum)

$$= f(0)\cdot\tfrac{\pi}{4} + f(\tfrac{\pi}{4})\cdot\tfrac{\pi}{4} + f(\tfrac{3\pi}{4})\cdot\tfrac{\pi}{4} + f(\pi)\cdot\tfrac{\pi}{4}$$

$$= 7.3939$$

(upper sum)

$$= f(\tfrac{\pi}{4})\cdot\tfrac{\pi}{4} + f(\tfrac{\pi}{2})\cdot\tfrac{\pi}{4} + f(\tfrac{\pi}{2})\cdot\tfrac{\pi}{4} + f(\tfrac{3\pi}{4})\cdot\tfrac{\pi}{4}$$

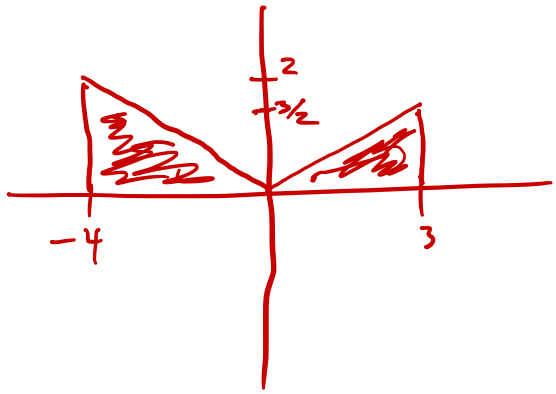
$$= 8.9647$$

3. Evaluate the integral by interpreting it in terms of areas. (*Hint: Start by sketching the integrand.*)

$$\int_{-4}^{3} \left|\tfrac{1}{2}x\right| dx = \int_{-4}^{0} |\tfrac{1}{2}x|\,dx + \int_{0}^{3} |\tfrac{1}{2}x|\,dx$$

$$= \tfrac{1}{2}\cdot 4\cdot 2 + \tfrac{1}{2}\cdot 3\cdot \tfrac{3}{2}$$

$$= 4 + \tfrac{9}{4} = \tfrac{25}{4}$$



4. (a) Set up an expression for the following integral as a limit of sums; you will not be able to compute the limit:

$$\int_0^5 \arctan x\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} \arctan(x_i)\,\Delta x$$

$$\left[\Delta x = \frac{5-0}{n} = \frac{5}{n},\quad x_i = 0 + i\cdot\frac{5}{n}\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \arctan\left(i\cdot\frac{5}{n}\right)\cdot\frac{5}{n}$$

(b) Using a graph of $y = \arctan x$, sketch a diagram which shows that

$$\frac{5\arctan(5)}{2} \le \int_0^5 \arctan x\, dx \le \frac{5\pi}{2}$$

green triangle: area $= \frac{1}{2}\cdot 5\cdot \arctan(5)$

red area

blue rectangle: $5\cdot\frac{\pi}{2}$ (area)