For each problem,

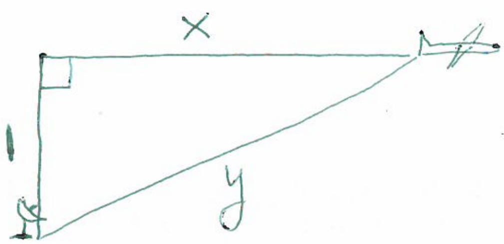
(a) Draw a sketch of the situation.

(b) Name (as variables) the quantities which are changing in time.

(c) Write an equation relating the (variable and constant) quantities.

(d) Finish solving the problem.

1. A plane flying horizontally at an altitude of 1 mile and a speed of 500 miles per hour passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station. } want $\frac{dy}{dt}$



$$1^2 + x^2 = y^2$$

$$\frac{dx}{dt} = 500 \frac{mi}{hr} \quad \text{(given)}$$

$$\frac{d}{dt}: \quad 0 + 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$x = \sqrt{y^2 - 1^2} = \sqrt{2^2 - 1} = \sqrt{3}$$

$$\iff \frac{dy}{dt} = \frac{x\frac{dx}{dt}}{y} = \frac{(\sqrt{3})\cdot 500}{2} = \sqrt{3}\cdot 250 \frac{mi}{hr}$$

2. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the radius is 5 cm.

$$S = 4\pi r^2 \qquad \frac{dS}{dt} = -1 \frac{cm^2}{min} \quad \text{(given)}$$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{dS/dt}{8\pi r}$$

$$\frac{dd}{dt} = 2\frac{dr}{dt} = 2\left(\frac{dS/dt}{8\pi r}\right) = 2\left(\frac{-1}{8\pi\cdot 5}\right) = \frac{-1}{20\pi} \frac{cm}{min}$$

3. The rate of change of atmospheric pressure P with respect to altitude h is proportional to P. (This assumes the temperature is constant.)

(a) Write a differential equation corresponding to the first sentence above; use k for the constant of proportionality. Then write a formula for $P(h)$ in terms of $P(0)$, k, and h.

$$\frac{dP}{dh} = kP \iff P(h) = P(0)e^{kh}$$

h=0

(b) At a temperature of $15\,°C$, the pressure is 101.3 kPa at sea level and the pressure is 87.14 kPa at $h = 1000$ m. From these facts, determine $P(0)$ and k.

$$P(0) = 101.3$$

$$87.14 = P(1000) = P(0)e^{k \cdot 1000} = 101.3e^{1000k}$$

$$\boxed{k = \ln\left(\frac{87.14}{101.3}\right)/1000 = -0.0001506\ \frac{1}{m}}$$

(c) What is the pressure at the top of Denali, at an altitude of 6187 m? (*This problem in the book, #19 in §3.8, has an error. It calls it "Mount McKinley."*)

$$P(6187) = 101.3e^{(-0.0001506)6187} = 39.905\text{ kPa}$$

(d) At what altitude is the pressure 1/3 of what it is at sea level?

$$\frac{1}{3}(\cancel{101.3}) = P(h) = (\cancel{101.3})e^{(-0.0001506)h}$$

$$\frac{\ln\left(\frac{1}{3}\right)}{-0.0001506} = h$$

$$h = 7295\text{ m}$$