

Recall the guidelines:

- domain
- intercepts
- symmetry
- asymptotes
- increase/decrease (and critical numbers)
- local maxima/minima
- concavity (and inflection points)
- sketch the graph

1. Sketch the graph by applying the guidelines:

$$y = \frac{1}{x^2 - 4}$$

A. $x \neq \pm 2$ or $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

B. y -intercept: $y = -\frac{1}{4}$, x -intercepts: none

C. even

D. $x = -2$ and $x = +2$ are vertical [e.g. $\lim_{x \rightarrow 2^+} \frac{1}{x^2 - 4} = +\infty$]
 $y = 0$ is horizontal [$\lim_{x \rightarrow \pm\infty} \frac{1}{x^2 - 4} = 0$]

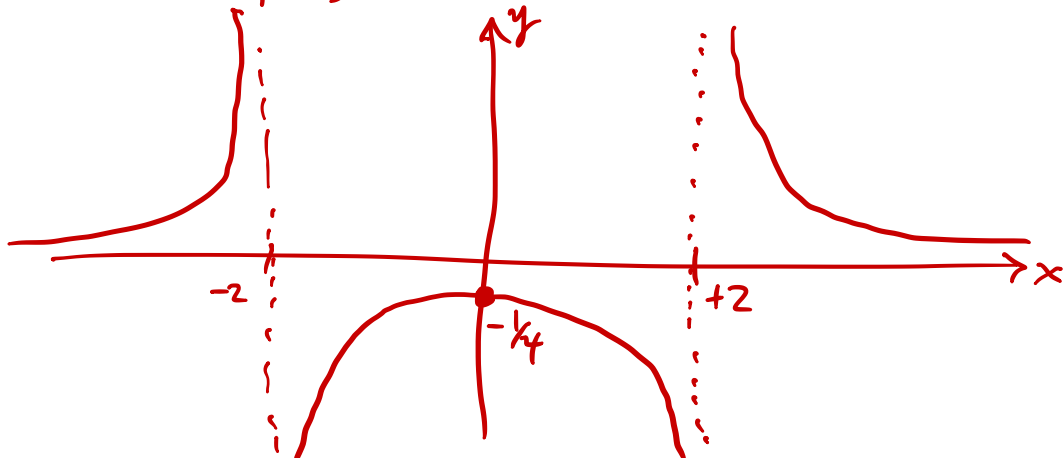
E. $y' = -(x^2 - 4)^{-2} (2x) = \frac{-2x}{(x^2 - 4)^2}$ $\therefore x = 0$ is crit. #

increasing $(-\infty, -2) \cup (-2, 0)$

decreasing $(0, 2) \cup (2, \infty)$

G. $y'' = \frac{-2(x^2 - 4)^{-2} - (-2x)2(x^2 - 4)(2x)}{(x^2 - 4)^4} = \frac{2(3x^2 + 4)}{(x^2 - 4)^3}$

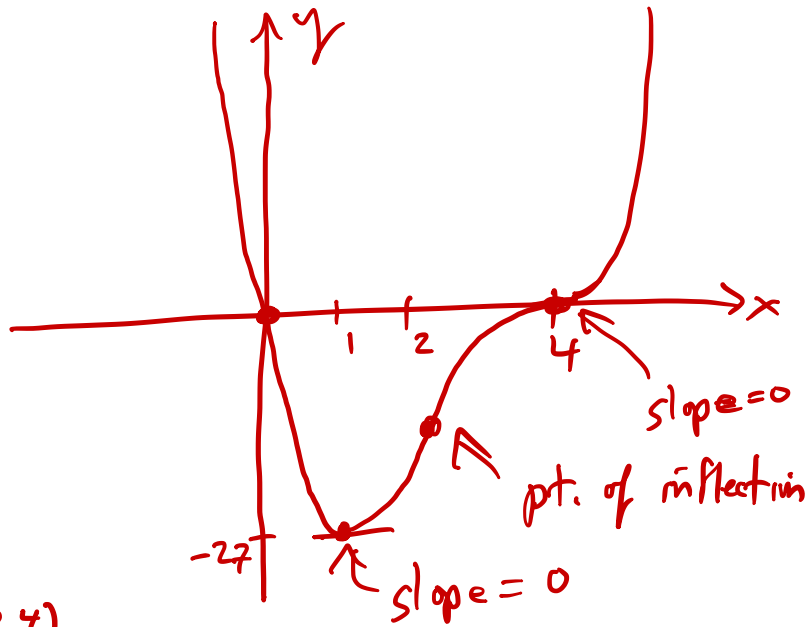
\therefore no inf. pts., concave up: $(-\infty, -2) \cup (2, \infty)$, concave down: $(-2, 2)$



2. Sketch the graph by applying the guidelines:

$$y = x(x-4)^3, \quad y' = 4(x-1)(x-4)^2, \quad y'' = 12(x-2)(x-4)$$

- A. $(-\infty, \infty)$
- B. $y=0, x=0$ & $x=4$
- C. none
- D. none
- E. increasing on $[1, \infty)$
decreasing on $(-\infty, 1]$
crit. #s: $x=1, 4$
- F. $x=1$ is loc. min. ($y=-27$)
no loc. max
- G. $x=2, 4$ inflection points
concave up: $(-\infty, 2) \cup (4, \infty)$, down: $(2, 4)$



3. Sketch the graph by applying the guidelines:

$$y = \frac{x}{\sqrt{x^2+1}}, \quad y' = \frac{1}{(x^2+1)^{3/2}}, \quad y'' = \frac{-3x}{(x^2+1)^{5/2}}$$

- A. $(-\infty, \infty)$
- B. $(0, 0)$ is both
- C. odd
- D. $y = -1, y = +1$ are hor. asymptotes
[e.g. $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}} = 1$]
- E. no crit. #s
increasing on $(-\infty, \infty)$
- F. $x=0$ is inf. pt.
concave up $(-\infty, 0)$
concave down $(0, \infty)$

