SOLUTIONS

1. Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2}$$

where G is the gravitational constant and r is the distance between the bodies.

(a) Find dF/dr and explain its meaning. What does the minus indicate?

$\frac{dF}{dr} = -2\frac{GmM}{r^3}$. this is the rate of change of force as the distance increases. the minus means the force is decreasing.

(b) Assume we measure mass in kilograms, distance in meters, and force in Newtons. What are the units of dF/dr?

units of $\frac{dF}{dr}$ are $\frac{\text{Newtons}}{\text{meter}}$

(c) Find dF/dm and explain its meaning and units.

units of $\frac{dF}{dm}$ are $\frac{\text{Newtons}}{\text{kilograms}}$

meaning: rate of change of force as one mass (m) increases

2. A tank holds 5000 gallons of water which drains from the bottom of the tank in 40 minutes. The volume of water remaining in the tank after t minutes is

$$V = 5000\left(1 - \frac{1}{40}t\right)^2$$

for $0 \le t \le 40$. Find the rate at which water is draining from the tank after (a) 5 min, (b) 20 min, and (c) 40 min. Which is fastest/slowest?

$V'(t) = 10000\left(1-\frac{1}{40}t\right)\left(-\frac{1}{40}\right) = -250\left(1-\frac{1}{40}t\right)$

(a) $V'(5) = -250\left(1-\frac{5}{40}\right) = -250\left(\frac{7}{8}\right) = -218.75 \frac{\text{gal}}{\text{min}}$ (fastest)

(b) $V'(20) = -250\left(1-\frac{20}{40}\right) = -250\left(\frac{1}{2}\right) = -125 \frac{\text{gal}}{\text{min}}$

(c) $V'(40) = -250\left(1-\frac{40}{40}\right) = 0 \frac{\text{gal}}{\text{min}}$ (slowest)

3. Differentiate the functions.

$y = \dfrac{1}{\log_3 x}$

$\dfrac{dy}{dx} = -(\log_3 x)^{-2}\left(\dfrac{1}{(\ln 3)\,x}\right)$

$y = \tan\left[\ln(ax+b)\right]$

$y' = \sec^2(\ln(ax+b)) \cdot \dfrac{1}{ax+b} \cdot a$

$H(z) = 7^z \arctan z$

$H'(z) = (\ln 7)\,7^z \arctan z + 7^z \dfrac{1}{1+z^2}$

$g(t) = \dfrac{\ln t}{\arcsin(t^2)+1}$

$g'(t) = \dfrac{\frac{1}{t}(\arcsin(t^2)+1) - \ln t\left(\frac{2t}{1+t^4}\right)}{(\arcsin(t^2)+1)^2}$