

# SOLUTIONS

## Math F251: Section 2.7 Worksheet

1. Find  $f'(a)$  using the definition of the derivative:

$$f(t) = 2t^2 + t$$

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{2(a+h)^2 + (a+h) - (2a^2 + a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{2a^2} + 4ah + 2h^2 + \cancel{a+h} - \cancel{2a^2} - \cancel{a}}{h} \\ &= \lim_{h \rightarrow 0} \frac{4ah + 2h^2 + h}{h} = \lim_{h \rightarrow 0} 4a + 1 + 2h = 4a + 1 \end{aligned}$$

2. Find  $f'(3)$  using the definition of the derivative:

$$f(x) = x^{-2}$$

$$\begin{aligned} f'(3) &= \lim_{h \rightarrow 0} \frac{(3+h)^{-2} - 3^{-2}}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{(3+h)^2} - \frac{1}{9}}{h} \\ &= \lim_{h \rightarrow 0} \frac{9 - (3+h)^2}{(3+h)^2 9 h} = \lim_{h \rightarrow 0} \frac{\cancel{9} - \cancel{9} - 6h - h^2}{(3+h)^2 9 h} \\ &= \lim_{h \rightarrow 0} \frac{-6h - h^2}{(3+h)^2 9 h} = \frac{-6}{3^2 \cdot 9} = \frac{-2}{27} \end{aligned}$$

3. Find  $f'(a)$  using the definition of the derivative:

$$f(x) = \sqrt{1+5x}$$

$$\begin{aligned} f'(a) &= \lim_{x \rightarrow a} \frac{\sqrt{1+5x} - \sqrt{1+5a}}{x-a} \quad \left( \begin{array}{l} \text{or:} \\ = \lim_{h \rightarrow 0} \frac{\sqrt{1+5(a+h)} - \sqrt{1+5a}}{h} \\ = \text{etc.} \end{array} \right) \\ &= \lim_{x \rightarrow a} \frac{\sqrt{1+5x} - \sqrt{1+5a}}{x-a} \cdot \frac{\sqrt{1+5x} + \sqrt{1+5a}}{\sqrt{1+5x} + \sqrt{1+5a}} \\ &= \lim_{x \rightarrow a} \frac{(1+5x) - (1+5a)}{(x-a)(\sqrt{1+5x} + \sqrt{1+5a})} = \lim_{x \rightarrow a} \frac{5(x-a)}{(x-a)(\sqrt{1+5x} + \sqrt{1+5a})} \\ &= \frac{5}{\sqrt{1+5a} + \sqrt{1+5a}} = \frac{5}{2\sqrt{1+5a}} \end{aligned}$$

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4. Find an equation of the tangent line to the curve at the given point:

$$f(x) = \frac{x+1}{x-1}, \quad (2, 3)$$

Also sketch both the curve  $y = f(x)$  and the tangent line.

$$m = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2}$$

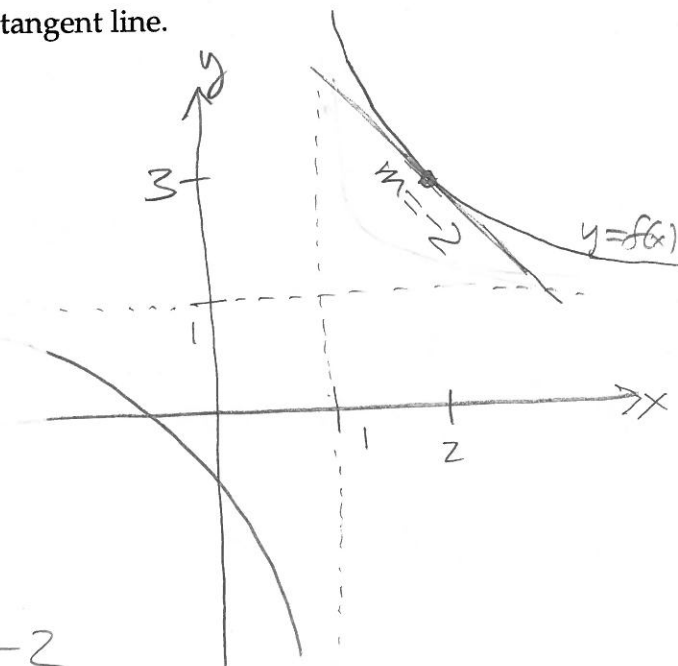
$$= \lim_{x \rightarrow 2} \frac{\frac{x+1}{x-1} - 3}{x-2}$$

$$= \lim_{x \rightarrow 2} \frac{(x+1) - 3(x-1)}{(x-1)(x-2)}$$

$$= \lim_{x \rightarrow 2} \frac{-2x + 4}{(x-1)(x-2)}$$

$$= \lim_{x \rightarrow 2} \frac{-2(\cancel{x-2})}{(x-1)(\cancel{x-2})} = \frac{-2}{1} = -2$$

$\therefore$  tangent line is  $y - 3 = (-2)(x - 2)$



5. A particle moves a distance  $s = f(t)$  along a straight line, where  $s$  is measured in meters and  $t$  is in seconds:

$$f(t) = 40t - 5t^2$$

Find the velocity and speed when  $t = 4$ .

$$v(4) = f'(4) = \lim_{h \rightarrow 0} \frac{40(4+h) - 5(4+h)^2 - 80}{h}$$

$$= \lim_{h \rightarrow 0} \frac{160 + 40h - 5(16 + 8h + h^2) - 80}{h}$$

$$= \lim_{h \rightarrow 0} \frac{160 + 40h - 80 - 40h - 5h^2 - 80}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-5h^2}{h} = \lim_{h \rightarrow 0} -5h = 0$$

$\therefore$  velocity = 0 m/s, speed = 0 m/s

$$f(4) = 80$$