SOLUTIONS

Math F251: Section 3.4 Worksheet

1. Find the derivative of the function. You do not need to simplify your answer.

(a) $y = \left(x + \dfrac{1}{x}\right)^7$

$$y' = 7\left(x + x^{-1}\right)^6 \cdot \left(1 - x^{-2}\right)$$

(b) $f(\theta) = \cos(\theta^2)$

$$f'(\theta) = -\sin(\theta^2) \cdot 2\theta$$

(c) $g(t) = 2^{(t^3)}$

$$g'(t) = (\ln 2)\, 2^{(t^3)} \cdot 3t^2$$

(d) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

$$\frac{dy}{dx} = \frac{1}{2}\left(x + \sqrt{x+\sqrt{x}}\right)^{-1/2}\left(1 + \frac{1}{2}\left(x+\sqrt{x}\right)^{-1/2} \cdot \left(1 + \frac{1}{2}x^{-1/2}\right)\right)$$

2. Find an equation of the tangent line to the curve at the given point.

$y = \sqrt{1+x^3}$, $(2,3)$

$$y' = \frac{1}{2}(1+x^3)^{-1/2}(0+3x^2)$$

$$y'(2) = \frac{1}{2}(1+2^3)^{-1/2}(3\cdot 2^2) = \frac{1}{\cancel{2}}\frac{1}{\sqrt{9}}\,3\cdot 2\cdot\cancel{2} = \frac{3\cdot 2}{3} = 2$$

$$y-3 = 2(x-2)$$

3. If $F(x) = f(g(x))$, and if $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, and $g'(5) = 6$, find $F'(5)$.

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by chain rule, $F'(x) = f'(g(x))\cdot g'(x)$

So

$$F'(5) = f'(g(5))\cdot g'(5)$$

$$= f'(\underbrace{-2}_{1})\cdot\underbrace{6}_{2} = \underbrace{4}_{3}\cdot 6 = 24$$

4. Find the 49th derivative of $f(x) = x\,e^{-x}$.

$$f'(x) = 1\cdot e^{-x} + x(-e^{-x}) = (1-x)e^{-x}$$

$$f''(x) = -1\cdot e^{-x} + (1-x)(-e^{-x}) = (-2+x)e^{-x}$$

$$f'''(x) = 1\cdot e^{-x} + (-2+x)(-e^{-x}) = (3-x)e^{-x}$$

So: $f^{(4)}(x) = (-4+x)e^{-x}$, $f^{(5)}(x) = (5-x)e^{-x}, \dots$

So $f^{(49)}(x) = (49-x)e^{-x}$