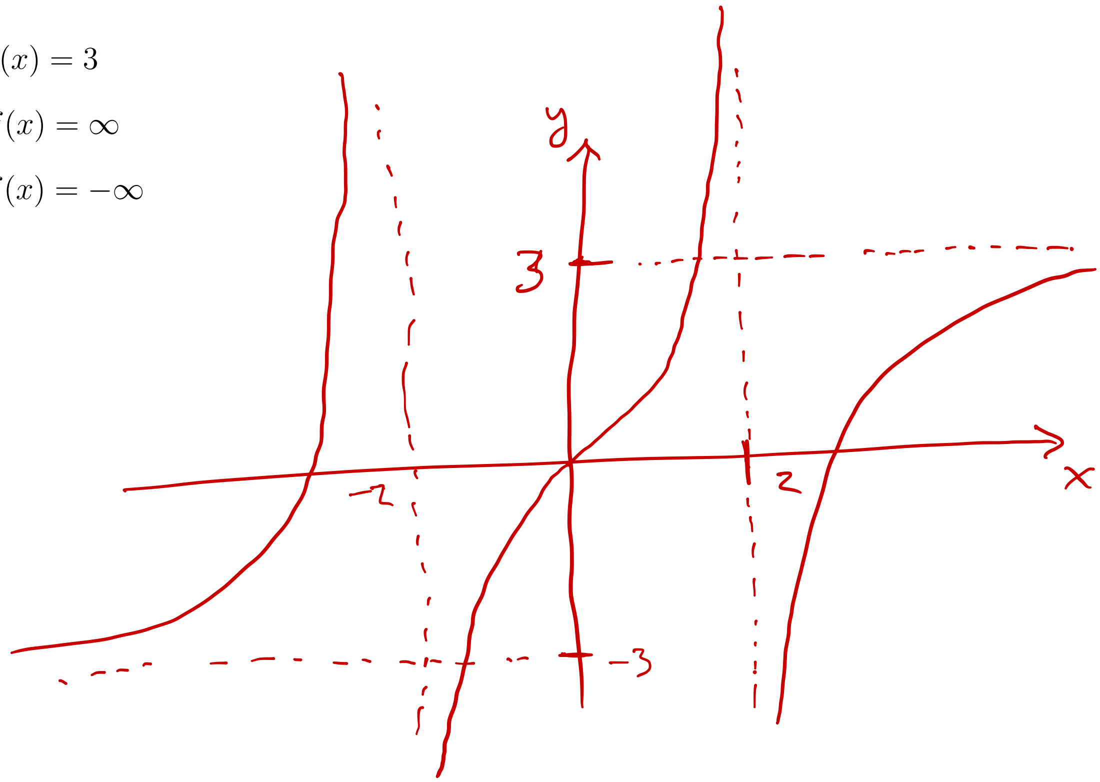
SOLUTIONS

1. Sketch the graph of a function that satisfies all of the given conditions:

- $\lim_{x\to\infty} f(x) = 3$
- $\lim_{x\to 2^-} f(x) = \infty$
- $\lim_{x\to 2^+} f(x) = -\infty$
- f is odd



2. Find all the vertical and horizontal asymptotes of the graph

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2},$$

and clearly state limits which justify these asymptotes. (*Also make a rough sketch of the graph. You may be able to confirm your work by graphing calculator.*)

$2x^2+x-1=0$

$x = \frac{-1 \pm \sqrt{1+8}}{4}$

$= -1, \frac{1}{2}$

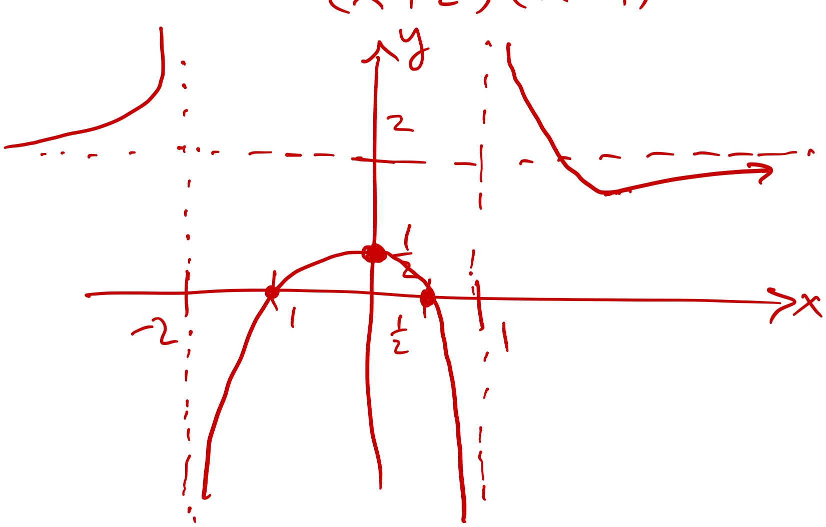
$$y = \frac{2x^2+x-1}{(x+2)(x-1)}$$

so: $\lim_{x\to\infty} \frac{2x^2+x-1}{(x+2)(x-1)} = 2 \therefore$ $y=2$ is hor.

$\lim_{x\to -2^+} \frac{2x^2+x-1}{(x+2)(x-1)} = -\infty$

$\lim_{x\to 1^+} \frac{2x^2+x-1}{(x+2)(x-1)} = +\infty$

$\therefore$ $x=-2$ and $x=1$ are vert.



3. Show that f is continuous on $(-\infty, \infty)$, and sketch the graph:

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}$$

if $x > \frac{\pi}{4}$, f is continuous because $\cos x$ is continuous

if $x < \frac{\pi}{4}$, f is continuous because $\sin x$ is continuous

at $x = \frac{\pi}{4}$: $f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$, $\lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{x \to \frac{\pi}{4}^-} \sin x = \frac{\sqrt{2}}{2}$,

and $\lim_{x \to \frac{\pi}{4}^+} f(x) = \lim_{x \to \frac{\pi}{4}^+} \cos x = \frac{\sqrt{2}}{2}$

4. Prove that the equation has at least one real root:

$$\ln x = 3 - 2x$$

(*A calculator can help find an accurate approximation, but this is not required!*)

$f(x) = \ln x - 3 + 2x$ domain: $(0, \infty)$

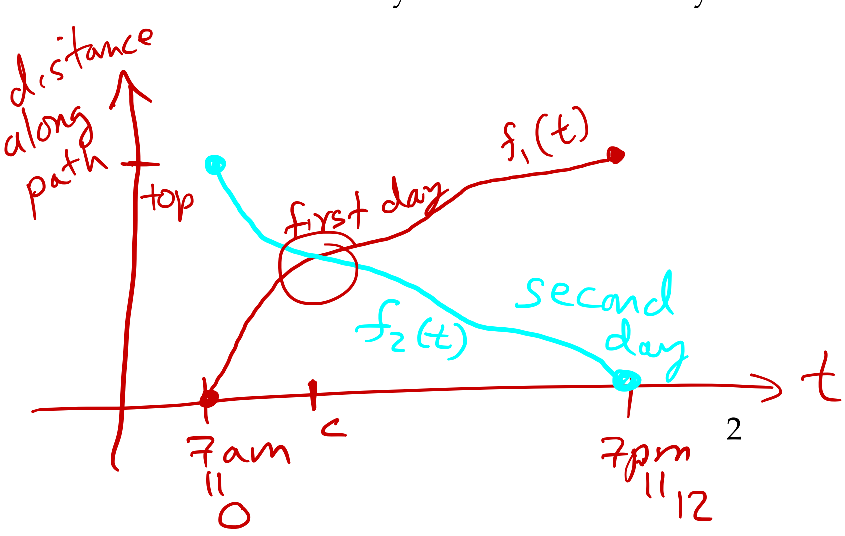
$f(1) = 0 - 3 + 2 = -1$

$f(e) = 1 - 3 + 2e > 0$ $[e \approx 2.7]$

since f is continuous, by IVT there is c in $(1, e)$ so that $f(c) = 0$

5. *A challenge problem, but actually easy. It follows from the Intermediate Value Theorem. Start by sketching elevation versus time for each day, one on top of the other.*

A Tibetan monk leaves the monastery at 7:00 AM and takes his usual path to the top of the mountain, arriving at 7:00 PM and sleeping on top. The next morning he starts at 7:00 AM at the top and takes the same path back, arriving at the monastery at 7:00 AM. Show that there is a point on the path that the monk will cross at exactly the same time of day on both days.



let $g(t) = f_1(t) - f_2(t)$

so $g(0) = 0 - (\text{top}) < 0$

$g(12) = (\text{top}) - 0 > 0$

since $g(t)$ is continuous (why? ... even monks don't teleport), there is c s.t. $g(c) = 0$

so $f_1(c) = f_2(c)$