SOLUTIONS

1. Find the derivative of $f(x) = (x + x^2)(x^{-1} + 3)$ in two ways:

(i) by the product rule:

$$f'(x) = (1+2x)(x^{-1}+3) + (x+x^2)(-x^{-2}+0)$$
$$= \cancel{x^{-1}}+3+2+6x+(-\cancel{x^{-1}}-1)$$
$$= 6x+4$$

(ii) by first expanding the product:

$$f(x) = 1+3x+x+3x^2 = 1+4x+3x^2$$
$$f'(x) = 4+6x \quad ✓$$

2. Differentiate.

(a) $y = \dfrac{\sqrt{x}}{2+x}$

$$\frac{dy}{dx} = \frac{\frac{1}{2}x^{-1/2}(2+x) - x^{1/2}(1)}{(2+x)^2}$$

Unsimplified is o.k. if derivative is final answer (and you are not asked to simplify)

(b) $g(x) = (\pi^{1/2} + 5\sqrt{x})\, e^x$

$$g'(x) = \left(0+\tfrac{5}{2}x^{-1/2}\right)e^x + (\pi^{1/2}+5\sqrt{x})e^x$$
$$= e^x\left(\pi^{1/2}+\tfrac{5}{2}x^{-1/2}+5x^{1/2}\right)$$

(c) $f(x) = \dfrac{ax+b}{cx+d}$

$$f'(x) = \frac{a(cx+d) - (ax+b)c}{(cx+d)^2}$$
$$= \frac{ad-bc}{(cx+d)^2}$$

3. If $h(2) = 4$ and $h'(2) = -3$, find

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2}$$

$$= \frac{h'(x)x - h(x)\cdot 1}{x^2}\Big|_{x=2} = \frac{h'(2)\cdot 2 - h(2)}{2^2}$$

$$= \frac{(-3)\cdot 2 - 4}{4} = \frac{-10}{4} = -5/2$$

4. Consider these facts:

- $\csc x = 1/\sin x$
- $\cot x = \cos x/\sin x$
- $(\sin x)' = \cos x$

Use the quotient rule and the above facts to show that

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\csc x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{0\cdot\sin x - 1\cdot\cos x}{(\sin x)^2}$$

$$= -\frac{1}{\sin x}\,\frac{\cos x}{\sin x} = -\csc x \cot x$$

5. Differentiate $f(\theta) = \theta\cos\theta\sin\theta$.

unsimplified o.k.!

$$f'(\theta) = 1\cdot\cos\theta\sin\theta + \theta\cdot(-\sin\theta)\sin\theta + \theta\cos\theta(\cos\theta)$$

$$= \cos\theta\sin\theta + \theta(\cos^2\theta - \sin^2\theta)$$

$$= \tfrac{1}{2}\sin(2\theta) + \theta\cos(2\theta) \longleftarrow$$

trig. identities:

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$