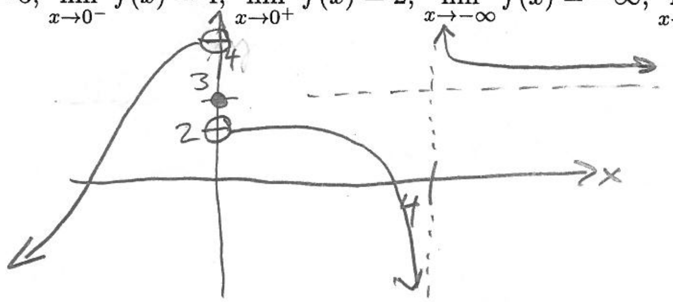
SOLUTIONS

1. (§2.6 #9) Sketch the graph of a function that satisfies all these conditions:

$$f(0)=3,\ \lim_{x\to 0^-} f(x)=4,\ \lim_{x\to 0^+} f(x)=2,\ \lim_{x\to -\infty} f(x)=-\infty,\ \lim_{x\to 4^-} f(x)=-\infty,\ \lim_{x\to 4^+} f(x)=\infty,\ \lim_{x\to\infty} f(x)=3$$



2. Find $f'(x)$ using the definition if $f(x)=\sqrt{x}$.

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to 0}\frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{1}{2\sqrt{x}}$$

3. (§2.7 #7) Using the result of the last problem, find an equation of the tangent line to $y=\sqrt{x}$ at the point $(1,1)$.

$$m=f'(1)=\frac{1}{2}$$

$$y-1=\frac{1}{2}(x-1)$$

4. (§2.6 #50) Find the horizontal and vertical asymptotes of the curve, and state the limits which justify these asymptotes:

$$y=\frac{1+x^4}{x^2-x^4}=\frac{1+x^4}{x^2(1-x)(1+x)}$$

horizontal: $y=-1$

vertial: $x=0, x=1, x=-1$

$$\lim_{x\to\infty}\frac{1+x^4}{x^2-x^4}=\lim_{x\to\infty}\frac{\frac{1}{x^4}+1}{\frac{1}{x^2}-1}=-1$$

$$\lim_{x\to 0^-}\frac{1+x^4}{x^2-x^4}=+\infty,\quad \lim_{x\to 1^-}\frac{1+x^4}{x^2-x^4}=+\infty,\quad \lim_{x\to -1^-}\frac{1+x^4}{x^2-x^4}=-\infty$$

5. (§2.3 #49) Let $g(x)=\frac{x^2+x-6}{|x-2|}$. (a) Find $\lim_{x\to 2^-} g(x)$ and $\lim_{x\to 2^+} g(x)$. (b) Does $\lim_{x\to 2} g(x)$ exist?

if $x>2$: $\quad g(x)=\frac{(x-2)(x+3)}{x-2}=x+3$

if $x<2$: $\quad g(x)=\frac{(x-2)(x+3)}{-(x-2)}=-x-3$

(a) $\lim_{x\to 2^-} g(x)=-2-3=-5$, $\quad \lim_{x\to 2^+} g(x)=5$

(b) $\lim_{x\to 2} g(x)$ d.n.e.