SOLUTIONS

Math F251: Sections 2.3 Worksheet

1. Sketch the graph $y = f(x)$ of a function which has all of the following properties; do not worry about any *formula* for $f(x)$:

(a) $f(0) = 3$

(b) $\lim_{x\to 0} f(x) = 0$

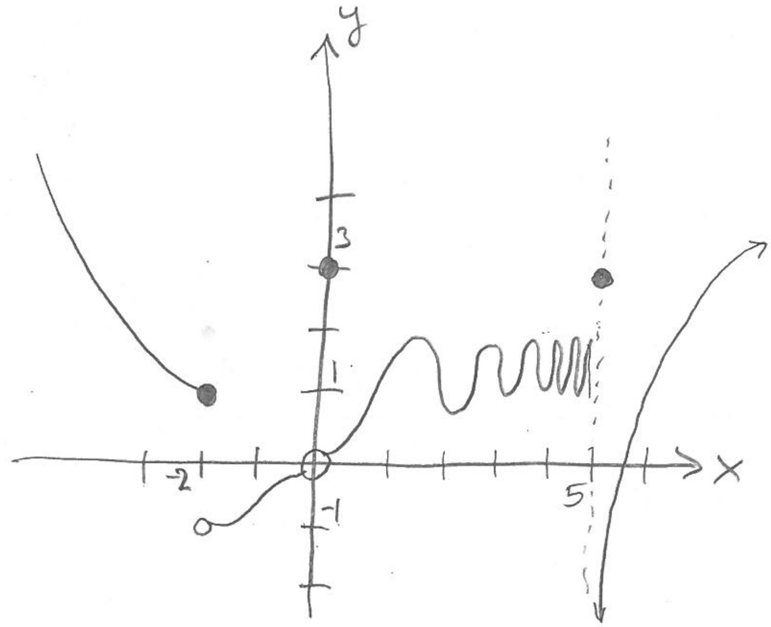
(c) $\lim_{x\to -2^-} f(x) = 1$

(d) $\lim_{x\to -2^+} f(x) = -1$

(e) $\lim_{x\to 5^-} f(x)$ d.n.e.

(f) $\lim_{x\to 5^+} f(x) = -\infty$

(g) the domain of f is $(-\infty, \infty)$



2. Evaluate the limit, if it exists:

$$\lim_{h\to 0} \frac{(2+h)^3 - 8}{h} = \lim_{h\to 0} \frac{\cancel{8} + 12h + 12h^2 + h^3 - \cancel{8}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(12+12h+h^2)}{\cancel{h}} = \lim_{h\to 0} 12 + 12h + h^2 = 12 + 0 + 0$$

$$= 12$$

3. Evaluate the limit, if it exists:

$$\lim_{u\to 2} \frac{\sqrt{4u+1} - 3}{u-2} = \lim_{u\to 2} \frac{\sqrt{4u+1} - 3}{u-2} \cdot \frac{\sqrt{4u+1} + 3}{\sqrt{4u+1} + 3}$$

$$= \lim_{u\to 2} \frac{(4u+1) - 9}{(u-2)(\sqrt{4u+1}+3)} = \lim_{u\to 2} \frac{4\cancel{(u-2)}}{\cancel{(u-2)}(\sqrt{4u+1}+3)}$$

$$= \lim_{u\to 2} \frac{4}{\sqrt{4u+1}+3} = \frac{4}{\sqrt{9}+3} = \frac{4}{6} = \frac{2}{3}$$

4. Evaluate the limit, if it exists:

$$\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}}-\frac{1}{t}\right) = \lim_{t\to 0}\frac{1-\sqrt{1+t}}{t\sqrt{1+t}} = \lim_{t\to 0}\frac{1-\sqrt{1+t}}{t\sqrt{1+t}}\cdot\frac{1+\sqrt{1+t}}{1+\sqrt{1+t}}$$

$$= \lim_{t\to 0}\frac{1-(1+t)}{t\sqrt{1+t}\,(1+\sqrt{1+t})} = \lim_{t\to 0}\frac{-\cancel{t}}{\cancel{t}\sqrt{1+t}\,(1+\sqrt{1+t})} = \frac{-1}{\sqrt{1}\,(1+\sqrt{1})} = \frac{-1}{2}$$

5. Evaluate the limit, if it exists:

$$\lim_{x\to 3}\frac{\frac{1}{x}-\frac{1}{3}}{x-3} = \lim_{x\to 3}\frac{3-x}{x\cdot 3\cdot(x-3)} = \lim_{x\to 3}\frac{-1}{3x} = \frac{-1}{9}$$

6. Evaluate the limits, if they exist, and otherwise explain why they do not:

(a)

$$\lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{|x|}\right) = -\infty$$

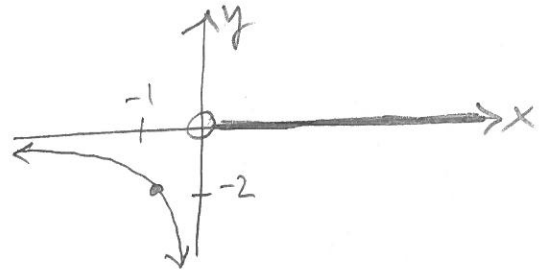
(b)

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{|x|}\right) = 0$$

Sketch via simplifications

$$\frac{1}{x}-\frac{1}{|x|}=0 \quad (x>0)$$

$$\frac{1}{x}-\frac{1}{|x|}=\frac{2}{x} \quad (x<0)$$



7. *Challenge problem.* Consider the following function:

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

Evaluate the limit $\lim_{x\to 0} f(x)$ if it exists. If it does not exist, explain why.

$\lim_{x\to 0} f(x)$ d.n.e.

because there is no number L so that $f(x)$ is close to L for all x close to zero