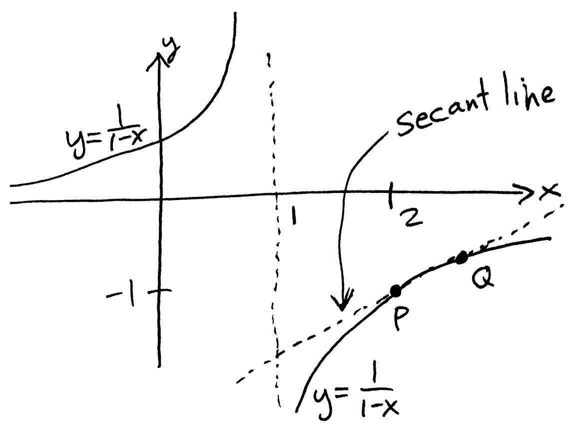
Solutions to Section 2.1 Activity (Worksheet)

1. *The first part has a clear answer. The second and third parts have best answers I could find. (A more thorough search might be required?) The fourth part has a guess.*

1. $m = (0.9 - 0.44)/(2017 - 1990) = +0.017$ C/year
2. the ten-year period $[1992, 2002]$: $m = (0.62 - 0.22)/10 = +0.04$ C/year
3. the ten-year period $[1998, 2008]$: $m = (0.52 - 0.62)/10 = -0.01$ C/year
4. from line through $(2010, 0.7)$ that looks vaguely reasonable: $m = +0.0074$ C/year

Every function, even data which is really bumpy or rough, does *have secant lines. It is always cheap to compute the slopes of secant lines, but for rough data such slopes don't mean much! However, calculus is mostly about better-behaved functions like in problems* **2** *and* **3**.

2. a)



b) *In this part P is the fixed point $(2, -1)$ while Q is the point given by the following x-values.* Let $f(x) = 1/(1-x)$. Then:

(i) $m = (f(1.5) - f(2))/(1.5 - 2) = 2.0$
(ii) $m = (f(1.9) - f(2))/(1.9 - 2) = 1.11$
(iii) $m = (f(1.99) - f(2))/(1.99 - 2) = 1.01$
(iv) $m = (f(1.999) - f(2))/(1.999 - 2) = 1.001$
(v) $m = (f(2.5) - f(2))/(2.5 - 2) = 0.6667$
(vi) $m = (f(2.1) - f(2))/(2.1 - 2) = 0.9091$
(vii) $m = (f(2.01) - f(2))/(2.01 - 2) = 0.9901$
(viii) $m = (f(2.001) - f(2))/(2.001 - 2) = 0.9990$

c) Based on the above I would guess that the tangent line slope at $P(2, -1)$ is $m = 1$.

d) We have a point and a slope so the equation of line is immediate:

$$y - (-1) = 1(x - 2) \qquad \text{or} \qquad y = x - 3$$

3. If you plot the data with a computer you see it is all close to one line. Here we first compute the four secant line slopes:

a) $m = 68.8$ beats/min
b) $m = 71.8$ beats/min
c) $m = 72.5$ beats/min
d) $m = 71.0$ beats/min

We can say with some confidence that the heart monitor should report a number between 69 and 72. My choice for best estimate averages the secant slopes from a) and b) and uses the difference for an estimate of uncertainty:

$$m = \frac{1}{2}(68.8 + 71.8) = 70.3 \pm 1.5 \text{ beats/min}$$