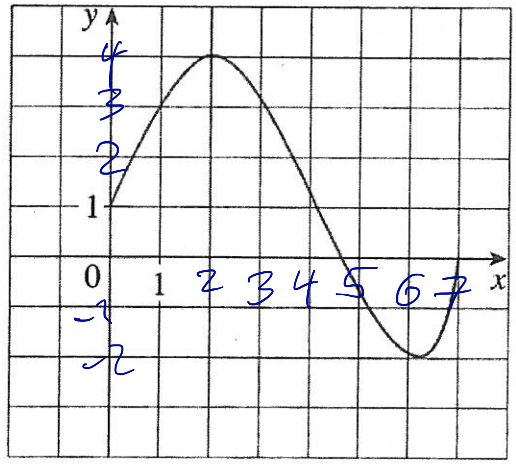
(SOLUTIONS)

Some approximation is necessary

1. The graph of a function f is shown below. Find the following:



a) $f(1)$ and $f(5)$ $f(1)=3$ $f(5)=-0.7$

b) the domain of f $[0,7]$

c) the range of f $[-2,4]$

d) For which value of x is $f(x) = 4$? $x=2$

e) Where is f increasing? $[0,2] \cup [6.2,7]$

2. Let $f(x) = 3x^2 - x + 2$. Find and simplify the following expressions.

(a) $f(2)$ $f(2) = 3\cdot 2^2 - 2 + 2 = 12$

(b) $f(a^2)$ $f(a^2) = 3a^4 - a^2 + 2$

(c) $[f(a)]^2$ $[f(a)]^2 = (3a^2 - a + 2)^2 = 9a^4 - 6a^3 + 13a^2 - 4a + 4$

(d) $\dfrac{f(2+h) - f(2)}{h}$

$$\frac{f(2+h)-f(2)}{h} = \frac{3(2+h)^2 - (2+h) + 2 - 12}{h} = \frac{\cancel{12} + 12h + 3h^2 \cancel{-2} - h \cancel{+2} \cancel{-12}}{h} = \frac{3h^2 + 11h}{h} = 3h + 11$$

(e) $\dfrac{f(a+h) - f(a)}{h}$

$$\frac{f(a+h)-f(a)}{h} = \frac{3(a+h)^2 - (a+h) + 2 - (3a^2 - a + 2)}{h}$$

$$= \frac{\cancel{3a^2} + 6ah + 3h^2 \cancel{-a} - h \cancel{+2} \cancel{-3a^2} \cancel{+a} \cancel{-2}}{h}$$

$$= \frac{(6a-1)h + 3h^2}{h} = 3h + 6a - 1$$

3. Find the domain of each of the following functions. Use interval notation.

1. $f(x) = \dfrac{1}{x^4 - 16}$

[Work: $x^4 - 16 = 0 \Leftrightarrow x^2 = 4$ or $x^2 = -4$ ← impossible; $x^2 = 4 \Leftrightarrow x = \pm 2$]

$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

2. $f(x) = \sqrt{x} + \sqrt{11 - x}$

[Work: $x \geq 0$ and $11 - x \geq 0 \Leftrightarrow x \leq 11$]

$[0, 11]$

3. $g(x) = \ln(x - 4)$

[Work: $x - 4 > 0 \Leftrightarrow x > 4$]

$(4, \infty)$

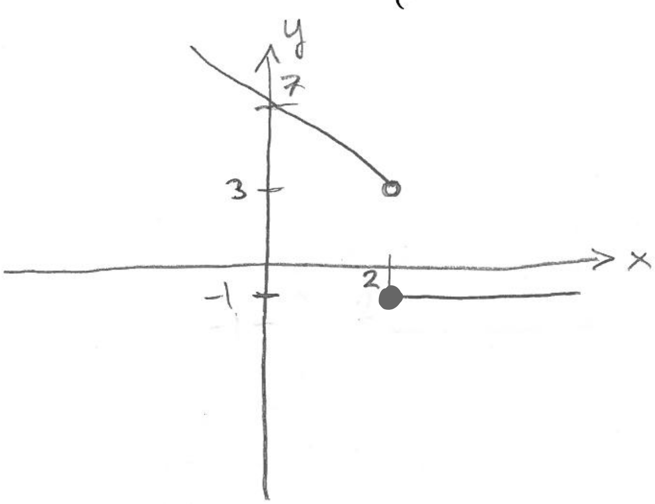
4. $h(x) = \frac{1}{\sqrt{x^2-5x-6}}$

[Work: $x^2 - 5x - 6 > 0$
$(x-6)(x+1) > 0$]

$(-\infty, -1) \cup (6, \infty)$

4. Graph each of the following piecewise defined functions.

a) $f(x) = \begin{cases} -1 & \text{if } x \geq 2 \\ 7 - 2x & \text{if } x < 2 \end{cases}$



b) $f(x) = \begin{cases} x + 1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$