SOLUTIONS

1. Differentiate the function.

$$F(r) = \frac{5}{r^3}$$

$$F(r) = 5r^{-3}$$

$$F'(r) = -15r^{-4}$$

$$y = 3e^x + \frac{4}{\sqrt[3]{x}}$$

$$y = 3e^x + 4x^{-1/3}$$

$$\frac{dy}{dx} = 3e^x - \frac{4}{3}x^{-4/3}$$

$$G(q) = (1 + q^{-1})^2$$

$$G(q) = 1^2 + 2q^{-1} + q^{-2} = 1 + 2q^{-1} + q^{-2}$$

$$G'(q) = -2q^{-2} - 2q^{-3}$$

2. Find equations of the tangent line and normal line to the curve at the given point: $y = x^2 + 2e^x$, $(0, 2)$

$$\frac{dy}{dx} = 2x + 2e^x$$

$$m_{tan} = \left.\frac{dy}{dx}\right|_{x=0} = 0 + 2e^0 = 2, \quad m_{normal} = \frac{-1}{2}$$

<u>tangent line:</u> $y - 2 = 2(x - 0)$

<u>normal line:</u> $y - 2 = \left(-\frac{1}{2}\right)(x - 0)$

3. The equation of motion of a particle is $s = t^4 - 2t^3 + t^2 - t$, where s is in meters and t is in seconds.

(a) Find the velocity and acceleration as functions of t.

$$v = s' = \dot{s} = 4t^3 - 6t^2 + 2t - 1$$

$$a = v' = s'' = \ddot{s} = 12t^2 - 12t + 2$$

(b) Find the acceleration after 1 s.

$$a(1) = 12 - 12 + 2 = 2 \ \frac{m}{s^2}$$

4. Find an equation of a tangent line to the curve $y = x^4 + 1$ which is parallel to the line $32x - y = 15$.

line $y = 32x - 15$ has $m = 32$

and $\frac{dy}{dx} = 4x^3 + 0 = 4x^3 \xrightarrow{\text{solve}} 4x^3 = 32$

$x = 2$

so $y = 2^4 + 1 = 17$

$$y - 17 = 32(x - 2)$$

5. Find the first and second derivatives of the function.

$G(r) = \sqrt{r} - \sqrt[3]{r}$

$$G(r) = r^{1/2} - r^{1/3}$$

$$G'(r) = \frac{1}{2} r^{-1/2} - \frac{1}{3} r^{-2/3}$$

$$G''(r) = -\frac{1}{4} r^{-3/2} + \frac{2}{9} r^{-5/3}$$