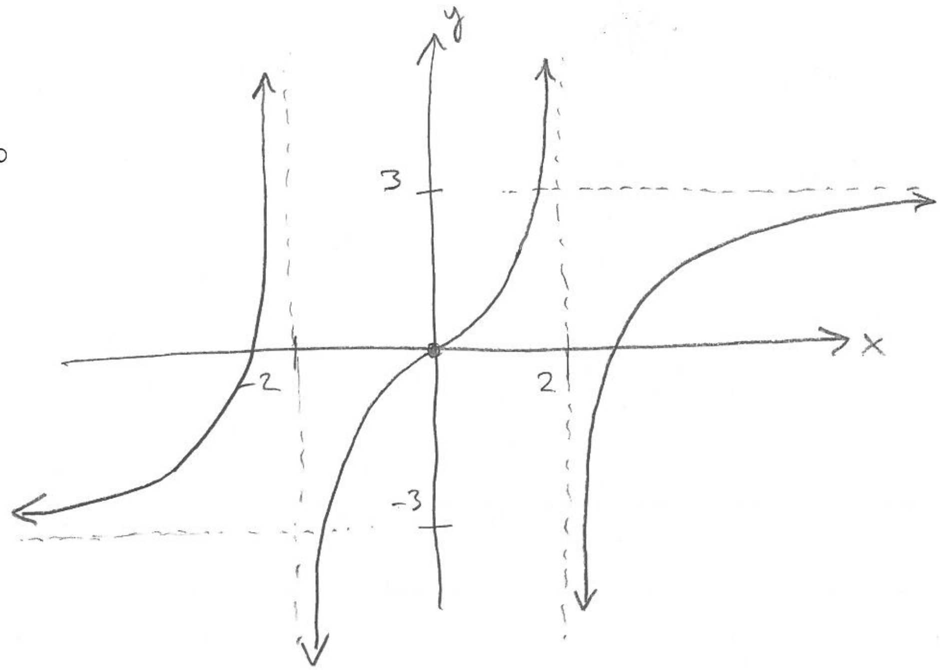
SOLUTIONS

Math F251: Sections 2.5 and 2.6 Worksheet 14 September 2018

there are other solutions!

1. Sketch the graph of a function that satisfies all of the given conditions:

- $\lim_{x\to\infty} f(x) = 3$
- $\lim_{x\to 2^-} f(x) = \infty$
- $\lim_{x\to 2^+} f(x) = -\infty$
- f is odd



2. Find all the vertical and horizontal asymptotes of the graph

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2},$$

and clearly state limits which justify these asymptotes.

(Along the way you may make a very rough sketch of the graph. You may be able to confirm your work by graphing calculator.)

vertical:

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2$ is vertical asymp.

$x = +1$ " " "

$$\lim_{x\to -2^-} \frac{2x^2+x-1}{x^2+x-2} = +\infty$$

$$\lim_{x\to +1^-} \frac{2x^2+x-1}{x^2+x-2} = -\infty$$

(note $2x^2 + x - 1 = (2x-1)(x+1)$ is not zero at $x=-2$ or $x=+1$)

horizontal

$$\lim_{x\to\infty} \frac{2x^2+x-1}{x^2+x-2} = \lim_{x\to\infty} \frac{2 + \frac{1}{x} - \frac{1}{x^2}}{1 + \frac{1}{x} - \frac{2}{x^2}} = \frac{2+0+0}{1+0+0} = 2$$

so $y = 2$ is (only) horizontal asymptote

3. Show that f is continuous on $(-\infty, \infty)$:

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}$$

note $\sin x$ and $\cos x$ are continuous everywhere

so we show: $\lim\limits_{x \to \frac{\pi}{4}} f(x) = f(\frac{\pi}{4})$ ✓

$\lim\limits_{x \to \frac{\pi}{4}^-} f(x) = \lim\limits_{x \to \frac{\pi}{4}^-} \sin(x) = \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

$\lim\limits_{x \to \pi/4^+} f(x) = \lim\limits_{x \to \frac{\pi}{4}^+} \cos(x) = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

$f(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

4. Prove that the equation has at least one real root:

$$\ln x = 3 - 2x$$

(You may use a calculator to find an accurate approximation, but this is not required.)

let $f(x) = \ln x - 3 + 2x$. We need to find x-values that give a change in sign:

$f(1) = \ln 1 - 3 + 2 \cdot 1 = 0 - 1 = -1$

$[f(\frac{1}{2}) = \ln(\frac{1}{2}) - 3 + 1 = \ln(\frac{1}{2}) + 1 = -\ln 2 + 1 < 0]$

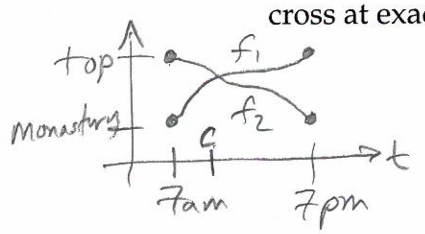
$f(2) = \ln 2 - 3 + 2 \cdot 2 = \ln 2 + 1 > 0$

So, because f is continuous on $(0, \infty)$, the IVT shows there is a solution on $(1, 2)$

5. *A challenge problem, but actually easy. It follows from the Intermediate Value Theorem. Start by sketching elevation versus time for each day, one on top of the other.*

A Tibetan monk leaves the monastery at 7:00 AM and takes his usual parth to the top of the mountain, arriving at 7:00 PM and sleeping on top. The next morning he starts at 7:00 AM at the top and takes the same path back, arriving at the monastery at 7:00 ~~AM~~. Show that there is a point on the path that the monk will cross at exactly the same time of day on both days.

pm



first day is $f_1(t)$, 2nd day is $f_2(t)$. let $g(t) = f_1(t) - f_2(t)$

$g(7am) < 0$, $g(7pm) > 0$

by IVT there is c so that $f_1(c) = f_2(c)$