"SOLUTIONS" = (my answers) ← yours should be different

Math F251 review: constructing possible final problems Friday 7 December 2018

1. (*Compare the examples and exercises in* §2.2 and §2.6.) Give an example of a graph $y = f(x)$ with a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = 2$.

$y = \dfrac{2x+7}{x+1}$ note $\lim_{x\to\infty} \dfrac{2x+7}{x+1} = 2$

$\therefore y = 2$ is hor. asymp.

so: $x+1 = 0$

$x = -1$ is vertical asymp.

2. (*Compare the examples and exercises in* §4.3 and §4.5.) In these sections there are problems which list a number of criteria for the graphs of functions, such as "$f(1) = 0$" or "$f'(x) > 0$ for $0 < x < 5$" or "f has an inflection point at $x = -3$". Build an example with 7 such criteria and sketch a graph with these properties. Can the criteria be in conflict?

criteria:

① $f(1) = 0$

② $f'(x) > 0$ for $0 < x < 5$

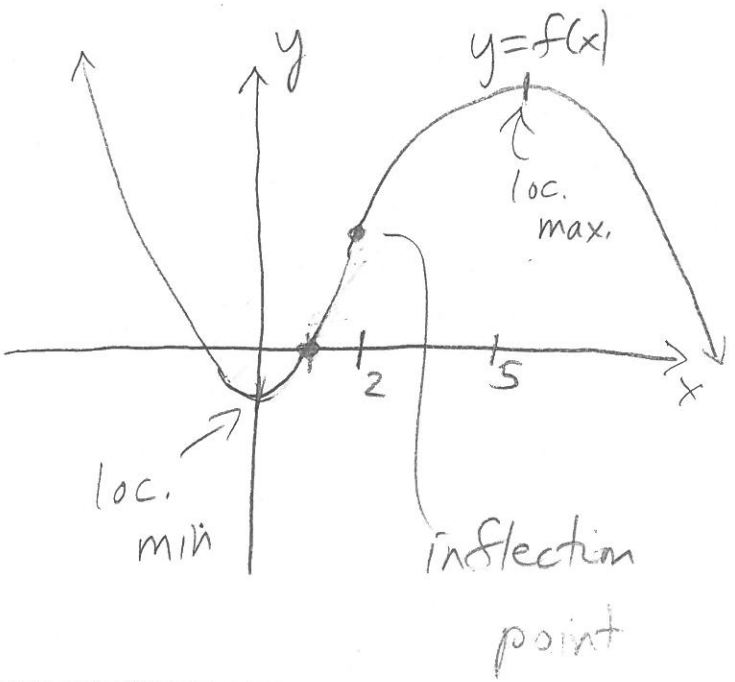
③ $f'(x) < 0$ for $x < 0$

④ $f'(x) < 0$ for $x > 5$

⑤ $f''(2) = 0$

⑥ $f''(x) > 0$ for $x < 2$

⑦ $f''(x) < 0$ for $x > 2$



Some advice for the actual Final Exam:

Read the question. Don't just guess it is of a certain type.

3. (*Compare the examples and exercises in* §3.4.) Build an example of a complicated chain rule derivative question. Compute the derivative.

Ex. $f(x) = 2^{\sin x + x^2}$, find $f'(x)$

Solution: $f'(x) = (\ln 2)\, 2^{\sin x + x^2} (\cos x + 2x)$

4. (*Compare the examples and exercises in* §5.5.) Write the previous example as an indefinite integration question. Give a substitution which will solve it, and do the integral. Is the problem you have built of reasonable difficulty?

Ex: Evaluate the integral: $\int 2^{\sin x + x^2} (\cos x + 2x)\, dx$

Soln: $\int 2^{\sin x + x^2} (\cos x + 2x)\, dx$ $\begin{cases} u = \sin x + x^2 \\ du = (\cos x + 2x)\, dx \end{cases}$

$= \int 2^u\, du = \frac{2^u}{\ln 2} + C = \frac{2^{\sin x + x^2}}{\ln 2} + C$

5. (*Compare the examples and exercises in* §3.9.) In related rates problems several quantities are changing in time, but these quantites are related by an equation. Build such an example.

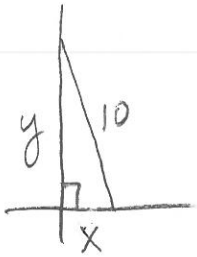
Ex: A ten-foot ladder is leaning against a wall. The base is slipping away from the wall at 2 feet per minute when the base is 3 feet from the wall. How fast is the top moving down the wall at that time?

Soln: $x^2 + y^2 = 10^2$

$3^2 + y^2 = 10^2$

$y = \sqrt{100 - 9} = \sqrt{91}$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$



$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -\frac{3}{\sqrt{91}} \cdot 2 \ \frac{ft}{min}$