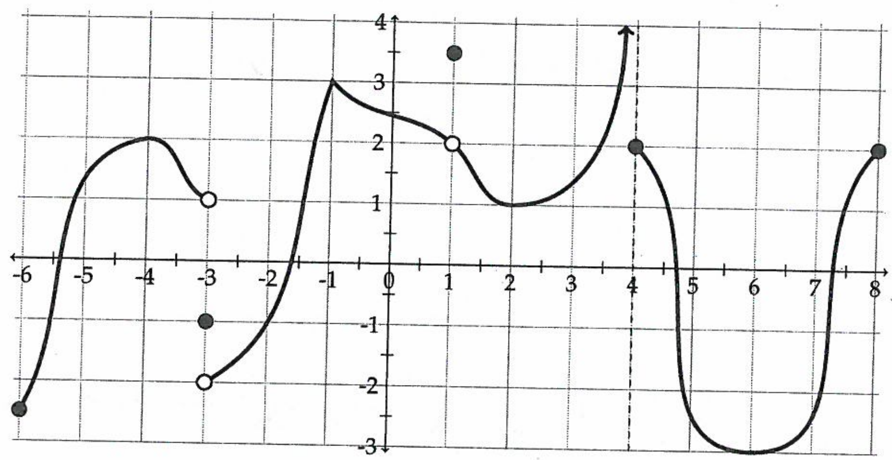
[this is a portion of the final in Fall2017... see uaf-github.io/exams.html for link to the solutions]

SOLUTIONS

$\boxed{1}$ (10 points) The graph of the function $f(x)$ is given below. Use it to answer the questions below. If you are asked to determine a limit, find the limit or one-sided limit as directed. Use ∞ and $-\infty$ where appropriate. If the limit does not exist and cannot be described using ∞ or $-\infty$, write "DNE".



(a) $\lim_{x\to -3^-} f(x) =$ 1

(b) $\lim_{x\to 1} f(x) =$ 2

(c) $\lim_{x\to -3} f(x) =$ DNE

(d) $\lim_{x\to 4^-} f(x) =$ $+\infty$

(e) At what x-values in its domain is $f(x)$ NOT continuous? If f is continuous everywhere on its domain, write "none".

$x = -3, 1, 4$

(f) At what x-values in its domain is $f(x)$ NOT differentiable? If f is differentiable everywhere on its domain, write "none".

$x = -6, -3, -1, 1, 4, 8$

(g) What are the x-values corresponding to local maxima of $f(x)$? If there aren't any, write "none".

$x = -4, -1, 1$

(h) What are the x-values corresponding to local minima of $f(x)$? If there aren't any, write "none".

$x = 2, 6$

(i) What are the x-values corresponding to absolute maxima of $f(x)$? If there aren't any, write "none".

none

(j) What are the x-values corresponding to absolute minima of $f(x)$? If there aren't any, write "none".

$x = 6$

2 (8 points)

(a) Complete the definition of the derivative of a function $f(x)$ below:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Find the derivative of $f(x) = 5x^2 - x$ using the definition of the derivative. You must show your work to receive credit.

$$f'(x) = \lim_{h \to 0} \frac{5(x+h)^2 - (x+h) - (5x^2 - x)}{h}$$

$$= \lim_{h \to 0} \frac{5(\cancel{x^2} + 2xh + h^2) \cancel{-x} - h \cancel{-5x^2} \cancel{+x}}{h}$$

$$= \lim_{h \to 0} \frac{10xh + 5h^2 - h}{h}$$

$$= \lim_{h \to 0} 10x + 5h - 1$$

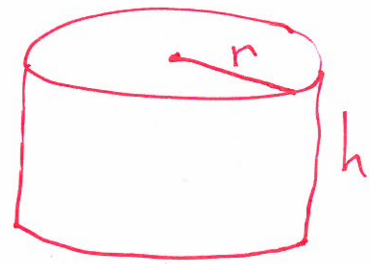
$$= 10x + 0 - 1 = 10x - 1$$

(which you know is right...)

3 (8 points) The volume of a circular cylinder is increasing at a rate of 20π m^3/sec while the radius is increasing at a rate of 2 m/sec. How must the height of the cylinder be changing when the volume is 90π m^3 and the radius is 3 m? Include units with your answer.

$V = \pi r^2 h$

V, r, h all changing



$$\frac{dV}{dt} = \pi\left(2r\frac{dr}{dt}\cdot h + r^2\frac{dh}{dt}\right)$$

know: dV/dt, $\frac{dr}{dt}$, V, r

for h: $90\pi = \pi\cdot 3^2 h \Rightarrow h = 10$

thus:

$$20\pi = \pi\left(2\cdot 3\cdot 2\cdot 10 + 3^2\cdot\frac{dh}{dt}\right)$$

$$20 = 120 + 9\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-100}{9}\ \frac{m}{s}$$

4 (15 points) Calculate the derivatives of the given functions.

(a) $y = (x^2+1)^{\cos x}$

$$\ln y = (\cos x) \ln(x^2+1)$$

$$\frac{1}{y} y' = -\sin x \ln(x^2+1) + (\cos x) \frac{1}{x^2+1} \cdot 2x$$

$$y' = (x^2+1)^{\cos x} \left(\frac{2x\cos x}{x^2+1} - (\sin x) \ln(x^2+1) \right)$$

(b) $g(z) = \dfrac{\sec(8z)}{1+z^2}$

$$g'(z) = \frac{8\sec(8z)\tan(8z)(1+z^2) - \sec(8z)(2z)}{(1+z^2)^2}$$

(c) $h(x) = \displaystyle\int_{\arctan x}^{5} \sqrt{3+2t^3}\,dt$ $= -\displaystyle\int_5^{\arctan x} \sqrt{3+2t^3}\,dt$

$\therefore$ by FTC I and chain rule,

$$h'(x) = -\sqrt{3+2(\arctan x)^3} \cdot \frac{1}{1+x^2}$$

5 (6 points) Let $f(x) = e^{4x} \cos x$.

(a) (4 points) Find the linearization of the function $f(x)$ at the point $a = 0$.

$$f'(x) = 4e^{4x}\cos x - e^{4x}\sin x$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= 1 + 4(x-0) = 1+4x$$

(b) (2 points)Use your linear approximation from part (a) to estimate $f(0.1)$.

$$f(0.1) \approx L(0.1) = 1+4(0.1) = 1.4$$

6 (6 points) Find the absolute maximum and absolute minimum of $f(x) = x^3 - 3x + 5$ on the interval $[0, 3]$.

$$f'(x) = 3x^2 - 3 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

x	$f(x)$
0	5
+1	4
3	23

abs. min

abs. max

7 (16 points) Answer the following questions using the given function and its derivatives. **Note that this problem continues onto the next page.**

$$f(x) = \frac{3x^2-1}{x^3}, \qquad f'(x) = \frac{-3(x^2-1)}{x^4}, \qquad f''(x) = \frac{6(x^2-2)}{x^5}$$

(a) Find the vertical asymptotes, if any.

$x = 0$ (because $\lim_{x \to 0^+} \frac{3x^2-1}{x^3} = -\infty$)

(b) Find the horizontal asymptotes, if any.

$y = 0$ (because $\lim_{x \to \infty} \frac{3x^2-1}{x^3} = 0$)

(c) Find the intervals of increase or decrease.

$f'(x)$ d.n.e. @ $x=0$

$f'(x) = 0$ @ $x = \pm 1$

$\therefore$ increase on $(-1,0) \cup (0,1)$

decrease on $(-\infty,-1) \cup (1,\infty)$

$f'(x)<0$ | $f'(x)>0$ | $f'(x)>0$ | $f'(x)<0$ → x

-1 | 0 | $+1$

(d) Find the local maximum and minimum values, if any.

$x = +1$ is local max.

$x = -1$ is local min

(e) Find the intervals of concavity and the inflection points.

$f''(x) = 0$

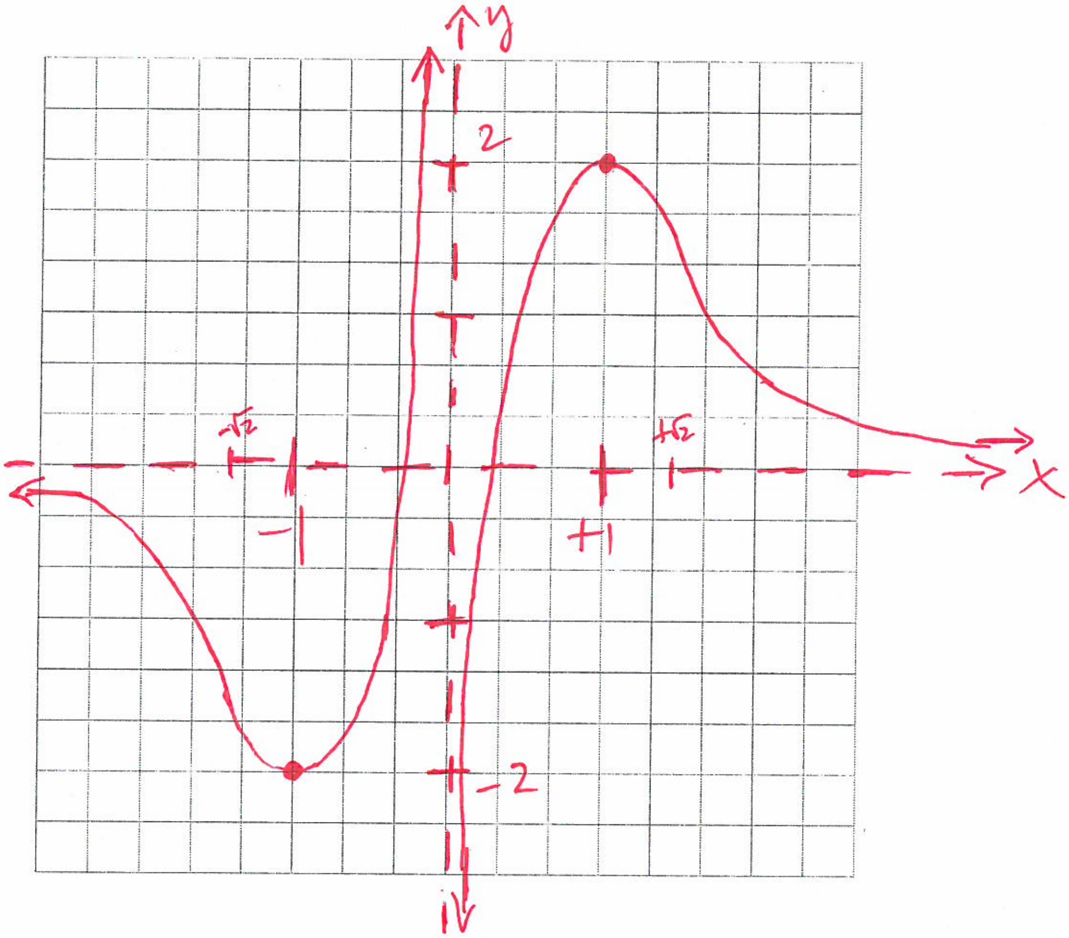
$\Leftrightarrow x^2 - 2 = 0$

$x = \pm\sqrt{2}$ ← inflection points

concave up on $(-\sqrt{2},0) \cup (\sqrt{2},\infty)$

concave down on $(-\infty,-\sqrt{2}) \cup (0,\sqrt{2})$

(f) Use the information from parts (a) - (e) to sketch the graph.



note:

$f(-1) = -2$

$f(+1) = +2$

f is odd