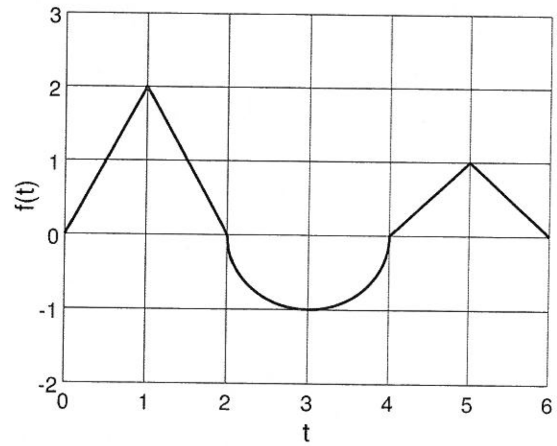
(SOLUTIONS)



**1.** (a) The graph of $f(t)$ is at right. Suppose we define the new function

$$g(x) = \int_0^x f(t)\,dt$$

Assuming the lines are straight and the curved part is circular, what are the exact values of $g(0), g(2), g(4), g(6)$?



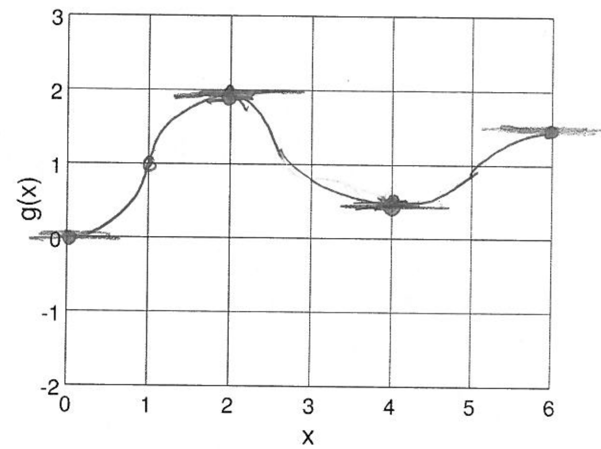

$g(0) = 0$

$g(2) = 2$

$g(4) = 2 - \pi/2$

$g(6) = 2 - \pi/2 + 1 = 3 - \pi/2$

(b) Sketch the graph of $g(x)$ on the provided axes.




(c) What is the graph of $g'(x)$?

Same as $f(x)$

**2.** (a) Use part I of the Fundamental Theorem of Calculus, and the chain rule, to find $dy/dx$ if

$$y = \int_{\cos x}^{\pi} \theta^2\,d\theta = -\int_{\pi}^{\cos x} \theta^2\,d\theta$$

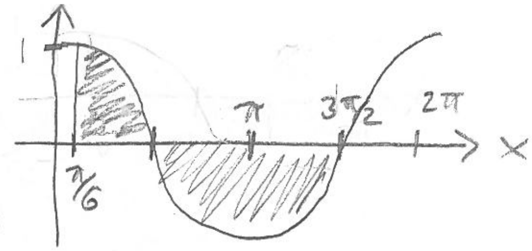
$$\therefore \frac{dy}{dx} = -\frac{d}{d\square}\left(\int_{\pi}^{\boxed{\cos x}} \theta^2\,d\theta\right) \cdot \frac{d\,\boxed{\cos x}}{dx} = -\cos^2 x(-\sin x)$$

$$= \sin x \cos^2 x$$

(b) Use part II of the Fundamental Theorem of Calculus to find $y = y(x)$. Then differentiate to find $dy/dx$ . . . and get the same result as in (a).

$$y = F(\pi) - F(\cos x) = \frac{\pi^3}{3} - \frac{1}{3}\cos^3 x \qquad \left(F(\theta) = \frac{\theta^3}{3}\right)$$

$$\frac{dy}{dx} = 0 - \frac{1}{3} \cdot 3\cos^2 x \cdot (-\sin x) = \sin x \cos^2 x$$




**3.** Evaluate the integral and interpret as a difference of areas:

$$\int_{\pi/6}^{3\pi/2} \cos x \, dx = \sin(3\pi/2) - \sin(\pi/6) = -1 - \frac{1}{2}$$

$F(x) = \sin x$

$$= -\frac{3}{2}$$

**4.** Evaluate the integral:

$$\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2} \, dx = F(\sqrt{3}) - F\left(\tfrac{1}{\sqrt{3}}\right) = 8\arctan(\sqrt{3}) - 8\arctan\left(\tfrac{1}{\sqrt{3}}\right)$$

$F(x) = 8 \arctan x$

$$= 8 \cdot \frac{\pi}{3} - 8 \cdot \frac{\pi}{6}$$

$$= 8 \cdot \frac{\pi}{6} = \frac{4\pi}{3}$$

**5.** Evaluate the integral:

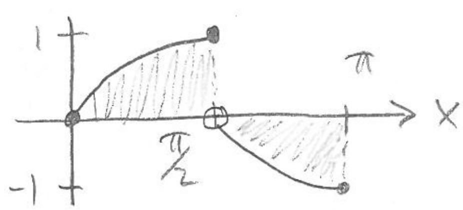
$$\int_0^1 (1+r)^3 \, dr = F(1) - F(0) = \frac{1}{4}(2)^4 - \frac{1}{4}(1)^4$$

$F(r) = \frac{1}{4}(1+r)^4$

$$= \frac{15}{4}$$

**6.** Suppose we define a function:

$$f(x) = \begin{cases} \sin x & \text{if } 0 \le x \le \pi/2 \\ \cos x & \text{if } \pi/2 < x \le \pi \end{cases}$$




Evaluate the integral $\int_0^\pi f(x)\,dx$.

$$\int_0^\pi f(x)\,dx = \int_0^{\pi/2} f(x)\,dx + \int_{\pi/2}^{\pi} f(x)\,dx$$

$$= \int_0^{\pi/2} \sin x \, dx + \int_{\pi/2}^{\pi} \cos x \, dx = -\cos(\pi/2) - (-\cos(0)) + \sin(\pi) - \sin(\pi/2)$$

$F(x) = -\cos x$ is antiderivative

$G(x) = \sin x$ is antiderivative

$$= -0 + 1 - 1 - 0$$

$$= 0$$