(SOLUTIONS)

Math F251: Sections 3.2 and 3.3 Worksheet — Monday 1 October 2018

1. Differentiate.

(a) $g(x) = (x + 5\sqrt{x})e^x$

$$g'(x) = (1 + 5 \cdot \tfrac{1}{2} x^{-\frac{1}{2}})e^x + (x + 5\sqrt{x})e^x \quad \leftarrow \text{product rule}$$
$$= (1 + \tfrac{5}{2}\tfrac{1}{\sqrt{x}} + x + 5\sqrt{x})e^x$$

(b)

$$y = \frac{\sqrt{x}}{2+x} \qquad \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}(2+x) - x^{\frac{1}{2}}(1)}{(2+x)^2} \quad \leftarrow \text{quotient rule}$$
$$= \frac{\frac{1}{2}\frac{1}{\sqrt{x}}(2+x) - \sqrt{x}}{(2+x)^2}$$

(c)

$$f(x) = \frac{ax+b}{cx+d} \quad \leftarrow \text{quotient rule}$$

$$f'(x) = \frac{a(cx+d) - (ax+b)c}{(cx+d)^2}$$
$$= \frac{acx + ad - acx - bc}{(cx+d)^2} = \frac{ad-bc}{(cx+d)^2}$$

2. Find the derivative in two ways: (i) product rule and (ii) first multiply-out.

$f(x) = (x + x^2)(x^{-1} + 3)$

(i) $f'(x) = (1+2x)(x^{-1}+3) + (x+x^2)(-x^{-2}+0)$
$= x^{-1} + 2 + 3 + 6x - x^{-1} - 1 = 4 + 6x$

(ii) $f(x) = 1 + x + 3x + 3x^2 = 1 + 4x + 3x^2$

$f'(x) = 4 + 6x$

3. A quantity p of fabric, measured in yards, is sold at a price $f(p)$ (dollars) which depends on the quantity. The total revenue from a sale of p yards of fabric is $R(p) = pf(p)$.

(a) What does it mean to say that $f(20) = 100$ and that $f'(20) = -0.5$?

"$f(20)=100$" means it costs \$100 to buy 20 yards of fabric

"$f'(20)=-0.5$" means the cost is dropping by 50¢ per yard (at 20 yards)

(b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

$R'(p) = 1 \cdot f(p) + p \cdot f'(p) = f(p) + p f'(p)$

So $R'(20) = 100 + 20(-0.5) = 100 - 12.5 = 87.5$

this is the rate of increase of the revenue as the number of yards increases (at 20 yards)

4. Consider these facts:

- $\csc x = 1/\sin x$
- $\cot x = \cos x/\sin x$
- $(\sin x)' = \cos x$

Use the quotient rule and the above facts to show that

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\csc x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{0 \cdot \sin x - 1 \cdot \cos x}{(\sin x)^2} = \frac{-\cos x}{\sin^2 x}$$

$$= -\frac{1}{\sin x}\frac{\cos x}{\sin x} = -\csc x \cot x$$

5. Differentiate $f(\theta) = \theta \cos\theta \sin\theta$.

$f'(\theta) = 1 \cdot \cos\theta \cdot \sin\theta + \theta(-\sin\theta)\sin\theta + \theta \cdot \cos\theta \cdot (\cos\theta)$ ← use $(uvw)' = u'vw + uv'w + uvw'$

$= \cos\theta\sin\theta + \theta(\cos^2\theta - \sin^2\theta)$