SOLUTIONS

Math F251: Sections 3.6 and 3.7 Worksheet — Friday 12 October 2018

1. Differentiate the functions.

$$y = \frac{1}{\log_3 x}$$

$y = (\log_3 x)^{-1}$

$$\frac{dy}{dx} = -(\log_3 x)^{-2} \cdot \frac{1}{(\ln 3)x}$$

$$= \frac{-1}{(\ln 3) \times (\log_3 x)^2}$$

$$y = \tan[\ln(ax+b)]$$

$$y' = \sec^2[\ln(ax+b)] \cdot \frac{1}{ax+b} \cdot a$$

$$= \frac{a \sec^2[\ln(ax+b)]}{ax+b}$$

$$g(t) = \frac{\ln t}{\arcsin(t^2)+1}$$

$$g'(t) = \frac{\left(\frac{1}{t}\right)(\arcsin(t^2)+1) - (\ln t)\left(\frac{1}{\sqrt{1-(t^2)^2}}\right)(2t)}{(\arcsin(t^2)+1)^2}$$

2. Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2} \quad = GmMr^{-2}$$

where G is the gravitational constant and r is the distance between the bodies.

(a) Find dF/dr and explain its meaning. What does the minus indicate?

$$\frac{dF}{dr} = -GmMr^{-3} = \frac{-GmM}{r^3}$$

rate of change of force with respect to radius

minus shows force decreases when r increases

(b) Assume we measure mass in kilograms, distance in meters, and force in Newtons. What are the units of dF/dr?

Newtons per meter $= \frac{N}{m}$

(c) Find dF/dm and explain its meaning and units.

$\frac{dF}{dm} = \frac{GM}{r^2}$, units $\frac{N}{kg}$, is rate of change of force with respect to (one) mass

3. **(CORRECTED!)** A tank holds 5000 gallons of water which drains from the bottom of the tank in 40 minutes. The volume of water remaining in the tank after t minutes is

$$V = 5000\left(1 - \frac{1}{40}t\right)^2$$

for $0 \le t \le 40$. Find the rate at which water is draining from the tank after (a) 5 min, (b) 20 min, and (c) 40 min. Which is fastest/slowest?

$$\frac{dV}{dt} = 10000\left(1 - \frac{1}{40}t\right)\left(-\frac{1}{40}\right)$$

$$= -250\left(1 - \frac{1}{40}t\right)$$

(a) -218.75 $\frac{\text{gallon}}{\text{min}}$ ← fastest

(b) -125 $\frac{\text{gallon}}{\text{min}}$

(c) 0 $\frac{\text{gallon}}{\text{min}}$ ← slowest

before CORRECTION:

$V = 5000\left(1 - \frac{1}{40}t^2\right)$

$\frac{dV}{dt} = -\frac{2 \cdot 5000}{40}t = -250t$

(a) -1250 $\frac{\text{gallon}}{\text{min}}$ ← slowest

(b) -5000 $\frac{\text{gallon}}{\text{min}}$

(c) -10000 $\frac{\text{gallon}}{\text{min}}$ ← fastest