Worksheet:
Area under curves by limits of sums of rectangles

Following the pattern in examples in section 5.2, we will do a problem in several steps:

Problem. Find the area under the parabola $f(x) = 4 - x^2$.

(a) Draw the area. What is the interval $[a, b]$? That is, what is the interval on which we want to find the area under $y = f(x)$?

(b) We will cut the interval $[a, b]$ (from part **(a)**) into n subintervals. What is Δx? What is a formula for x_i, the ith endpoint of the subintervals, where $x_0 = a$ and $x_n = b$?

(c) I flipped a coin and decided to use left endpoints. Using sigma notation, write a sum for $s(n)$, the total area of the n rectangles using left endpoints to evaluate $f(x)$.

(d) Simplify the sum $s(n)$ until you recognize one or more of the known sums in Theorem 5.2, page 296.

(e) Use the known sums from Theorem 5.2 to eliminate all the symbols "Σ".

(f) Find the limit as $n \to \infty$, that is, $A = \lim_{n\to\infty} s(n)$. This is the area under the curve.

Now compare your work to Examples 3, 4, and 6 in Section 5.2, all of which are areas under parabolas like this Problem. You should be able to recognize the steps used in these examples. Note that Example 6 the book is more brief: It starts from the statement

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

and expand and simplify (i.e. all the steps above) in just one calculation. Once you know what you are doing, this is what you should do.