Study Guide for Midterm Exam II

The Exam is in-class on Monday 11 November, 2013.
It is closed-book and closed notes. No calculators are allowed.

My summary of the *ideas* in the sections covered on Midterm Exam II:

3.4 The chain rule is the key to differentiating any combination of functions:

$$\Big[f\left(g(x)\right)\Big]' = f'\left(g(x)\right) g'(x) \qquad \text{or} \qquad \frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}.$$

Practice looking at complicated functions and seeing them as an outer function applied to an inner function. The "Basic Differentiation Rules" table in the inside front (or back) cover of your book states *all* the rules using the chain rule, such as

$$\frac{d}{dx}\left[\sin u\right] = \cos u\, u' \qquad \text{or} \qquad \frac{d}{dx}\left[u^n\right] = nu^{n-1}\, u'.$$

In section 3.4 we get the derivative rules for all exponentials and logarithms:

$$[e^x]' = e^x, \qquad [\ln x]' = \frac{1}{x}, \qquad [a^x]' = (\ln a)\, a^x, \qquad [\log_a x]' = \frac{1}{(\ln a)x}.$$

3.5 Implicit differentiation. (*Review this! Do some problems!*) A major application of the chain rule. If you think "y is a function of x even though I don't know it explicitly" and you use the chain rule then you'll see dy/dx appear everywhere there is a y in the equation. Solving for dy/dx is always easy because it appears as a factor and to the first power only.

3.6 We get a derivative rule for the inverse function $y = g(x)$ corresponding to the function f by using implicit differentiation on $f(y) = x$. This gives six new rules for inverse trig functions, of which you should memorize the rules for $\arcsin$, $\arctan$, arcsec and remember that the other three have negatives.

3.7 "Related rates" means differentiating an equation with respect to time t. In practice the hard part is writing down the equation. In each problem you must be clear on which variables change in time and which are constant. At the end of each problem you'll plug in numbers that only apply at an instant. (*Review this! Do some problems!*)

3.8 Newton's method. Know *what* problem it solves (i.e. it solves $f(x) = 0$) and *how* to do a step or two. Know *why* the equation "$x_{n+1} = x_n - f(x_n)/f'(x_n)$" makes sense as a way to improve a guess of the solution of $f(x) = 0$; i.e. draw the picture.

4.1 Extrema = (maximum or minimum). Fundamental ideas here: (i) a continuous function on a closed interval has both a maximum and a minimum, and (ii) you can find these extrema by listing the endpoints and the critical numbers and evaluating the function at this list.

4.2 The Mean Value Theorem says that the slope of the secant line between two points $x = a$ and $x = b$ is achieved by a tangent line somewhere in between (i.e. $x = c$). Rolle's Theorem is just the zero-slope case.

4.3 *Draw a number line* or build a table to organize a calculation from this or the next section. Of course, increasing goes with $f'(x) > 0$ and decreasing with $f'(x) < 0$. Graphs can switch from increasing to decreasing and vice versa both at critical numbers and at vertical asymptotes. *First derivative test*: You can decide whether a critical number is a relative minimum, relative maximum, or neither by evaluating $f'(x)$ *on either side of* the critical number.

4.4 Concave up goes with $f''(x) > 0$ and concave down with $f''(x) < 0$. Points of inflection are where the concavity switches, and so either $f''(x) = 0$ or $f''(x)$ does not exist at a point of inflection. *Second derivative test*: You can decide whether a critical number is a relative minimum, relative maximum, or neither by evaluating $f''(x)$ *at* the critical number.

4.5 Limits "at" infinity: know the informal definition and know the "divide both numeration and denominator by the highest power of x" tool for evaluating many of these limits. "Horizontal asymptote" is defined in this section. Make sure to do some examples with transcendental functions (e.g. $\cos x$ or e^{-x} or etc.) too.

4.6 These problems combine material in sections 4.3–4.5 and more. Though you should be able to choose the tools for sketching by yourself—you should have an "action plan" for sketching—on the midterm I'll break the problem into pieces. For instance I might say "find intercepts and vertical and horizontal asymptotes" for part (a) and then "sketch the graph identifying the relative extrema" in part (b).

Colored boxes to notice: The textbook has several colored boxes with "guidelines" or "basic differentiation rules" or such. Here are the ones I would focus on if I were you.

- section 3.6, page 179: "Basic differentiation rules . . . ". This is the list you should memorize, or at least be very comfortable with. Note that the rules include the chain rule, so u' appears on the right of most rules.
- section 4.1, page 207: "Guidelines for finding extrema on a closed interval". This is not just good advice, it's a theorem. Yes, follow this recipe in the appropriate problems. *And* understand why it comes from Theorems 4.1 and 4.2.
- section 4.6, page 249: "Guidelines for analyzing the graph of a function". You don't have to memorize this box, but you should own on your own "action plan" for addressing a "sketch and analyze the graph of this function" problem. Say your action plan out loud? Be flexible when applying your action plan.

In addition, on pages 167, 183, 220, and 241 there are "Guidelines . . . " boxes. My advice is *not* to memorize these, but to refer to them as you do the corresponding problems. These boxes should *make sense to you*; that is more important than memorizing.

Remember to:

. . . *read the question*. Are you doing an operation which addresses the question?
. . . *use "=" like you mean it*. Are the objects on either side of "=" actually equal?