Study Guide for Final Exam

The Final Exam is in Gruening 408 (*not* the usual room!)
at 10:15–12:15 on Tuesday, 17 December, 2013.

It is closed-book and no calculators are allowed,
BUT YOU MAY BRING 1/2 SHEET OF LETTER PAPER AS NOTES.

The Final Exam will have about half RECENT and
half COMPREHENSIVE material. These portions will be clearly marked.

My review summary of the ideas in the RECENT sections:

4.7 *Review this! Do some problems!* Optimization. In these word problems, your goal is to set up a function of one variable. The function should output the quantity you want to make largest or smallest. Take the derivative and set to zero, but also think about the domain and the endpoints or the limits (for the input variable).

4.8 The only thing we did in this section is a gloss of tangent line approximations. Be able to do exercises 1 through 6. Note $T(x) = f(c) + f'(c)(x-c)$ is the tangent line at $(c, f(c))$.

5.1 The basic idea is just notation: indefinite integral $\int f(x)\,dx$ means the general antiderivative of $f(x)$. The antiderivative question can be stated as a differential equation. For indefinite integrals one writes "$+C$". Know how to find the constant if an initial value is given for a differential equation.

5.2 Practice the easy problems "use sigma notation to write the sums". Practice the harder problems "use the summation formulas to rewrite the sum without sigma". Memorize the summation formulas for $\sum_{i=1}^n c$, $\sum_{i=1}^n i$, $\sum_{i=1}^n i^2$ (see page 296). I'll give you the formula for $\sum_{i=1}^n i^3$ if needed. Use $\Delta x = (b-a)/n$ and $x_i = a + i\Delta x$ to set up the sums which approximate areas. Practice the limit process to compute areas: do problems in the range 57–66.

5.3 Know the limit definition of the definite integral on page 309:

$$\int_a^b f(x)\,dx = \lim_{\|\Delta\|\to 0} \sum_{i=1}^n f(c_i)\,\Delta x_i.$$

What do all these symbols mean? What part of this definition is the "Riemann sum"? Understand when the definite integral is, and is not, simply the area under the graph $y = f(x)$. Do problems where you compute the definite integral by adding and/or subtracting areas you get from geometric formulas.

5.4 This section is the most important in Chapter 5. It says that if we want to find areas (definite integrals) then we should just find an antiderivative and plug numbers into it. Understand both forms of the Fundamental Theorem of Calculus (FTC). The Second FTC says that derivative and integral are inverse operations; practice some problems using the Second FTC. Review definition of average value of a function.

5.5 "Substitution" means "find $u = g(x)$ so that $g'(x)$ is also present so that $\int f(g(x))g'(x)\,dx = \int f(u)\,du$". For definite integrals the limits of integration can be changed from values of x to values of u so that you don't have to "go back". (*Practice substitution problems!*)

5.6 Know how to do simple $n = 2$ or $n = 4$ Trapezoid rule and Simpson's rule problems. What geometric picture yields Simpson's rule?

5.7 More substitutions of a certain type: $u = g(x)$ is the denominator, so that you use $\int du/u = \ln|u| + C$. Allows us to integrate $\tan x$, for instance.

5.8 Three more integrals to know: see page 363. Practice problems from this section to see what forms turn into the three integrals after a substitution. Know all the rules in the colored box on page 366.

Colored boxes in RECENT sections: The textbook has several colored boxes with "guidelines" or "basic rules" or such. Here are the ones I would focus on if I were you.

- section 4.7, page 260: This is good advice–do understand it!–but don't bother memorizing. Do example problems.
- section 5.1, page 286: This box makes an obvious point, that for each differentiation rule there is a corresponding antiderivative (i.e. integration) rule. Memorize the table on page 366 instead of this one.
- section 5.4, page 319: This is good advice which you should be following already.
- section 5.5, page 337: This is good advice but it forgets a critical recommendation: choose $u = g(x)$ so that $g'(x)$ is *essentially* (say, up to a constant) already present in the integrand. There is no way to "get" substitution without practice!
- section 5.7, page 357: This is good advice, which you should be following already. Still, it is not guaranteed to work on all integrals.
- section 5.7, page 359: Memorize the table on page 366 instead of this one.
- section 5.8, page 366: *Memorize this one!* And notice it is analogous to the table of "Basic differentiation rules" on page 179.

Reviewing for the COMPREHENSIVE part of the Final Exam:

(1) Look at the previous Study Guides for Midterms I and II. (*They are online as PDFs.*)

(2) Look at your quizzes and their solutions. What kinds of problems kept reappearing?

(3) Go though Chapters 2, 3, 4 which we have not done recently. Focus on the major topics and types of problems:
- the definition of the limit: *expect "write the definition of* $\lim_{x\to c} f(x) = L$ *either as a sentence or using the* ϵ–δ *definition"*
- the definition of continuity: *expect "define* $f(x)$ is continuous at $x = c$"
- the definition of the derivative (as a limit): *expect "define* $f'(x)$"
- the meaning of the derivative as the slope of the tangent line: *expect a problem of the type "find the tangent line"*
- practice product, quotient, and chain rules
- practice implicit differentiation problems
- practice related rates problems
- practice finding extrema on closed intervals
- practice curve sketching using derivatives: critical points, intervals of increase/decrease, concavity, inflection points, tests for relative extrema; *do problems from 4.6*
- practice optimization problems

(4) Yes, I'll put one or two "major ideas only" problems from Chapter 5 on the COMPREHENSIVE part, with the idea that these will be easy points.